On the generalized Freedman-Townsend model

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Abstract

Consistent interactions that can be added to a free, Abelian gauge theory comprising a finite collection of BF models and a finite set of two-form gauge fields (with the Lagrangian action written in first-order form as a sum of Abelian Freedman-Townsend models) are constructed from the deformation of the solution to the master equation based on specific cohomological techniques. Under the hypotheses of smoothness in the coupling constant, locality, Lorentz covariance, and Poincaré invariance of the interactions, supplemented with the requirement on the preservation of the number of derivatives on each field with respect to the free theory, we obtain that the deformation procedure modifies the Lagrangian action, the gauge transformations as well as the accompanying algebra. The interacting Lagrangian action contains a generalized version of non-Abelian Freedman-Townsend model. The consistency of interactions to all orders in the coupling constant unfolds certain equations, which are shown to have solutions.

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1 Introduction

The power of the BRST formalism was strongly increased by its cohomological development, which allowed, among others, a useful investigation of

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many interesting aspects related to the perturbative renormalization problem [1, 2, 3, 4, 5], anomaly-tracking mechanism [5, 6, 7, 8, 9, 10], simultaneous study of local and rigid invariances of a given theory [11] as well as to the reformulation of the construction of consistent interactions in gauge theories [12, 13, 14, 15, 16] in terms of the deformation theory [17, 18, 19] or, actually, in terms of the deformation of the solution to the master equation [20, 21].

The scope of this paper is to investigate the consistent interactions that can be added to a free, Abelian gauge theory consisting of a finite collection of BF models and a finite set of two-form gauge fields (described by a sum of Abelian Freedman-Townsend actions). Each BF model from the collection comprises a scalar field, a two-form and two sorts of one-forms. We work under the hypotheses that the interactions are smooth in the coupling constant, local, Lorentz covariant, and Poincaré invariant, supplemented with the requirement on the preservation of the number of derivatives on each field with respect to the free theory. Under these hypotheses, we obtain the most general form of the theory that describes the cross-couplings between a collection of BF models and a set of two-form gauge fields. The resulting interacting model is accurately formulated in terms of a gauge theory with gauge transformations that close according to an open algebra (the commutators among the deformed gauge transformations only close on the stationary surface of deformed field equations).

Topological BF models [22] are important in view of the fact that certain interacting, non-Abelian versions are related to a Poisson structure algebra [23] present in various versions of Poisson sigma models [24, 25, 26, 27, 28, 29, 30], which are known to be useful at the study of two-dimensional gravity [31, 32, 33, 34, 35, 36, 37, 38, 39, 40] (for a detailed approach, see [41]). It is well known that pure three-dimensional gravity is just a BF theory. Moreover, in higher dimensions general relativity and supergravity in Ashtekar formalism may also be formulated as topological BF theories with some extra constraints [42, 43, 44, 45]. Due to these results, it is important to know the self-interactions in BF theories as well as the couplings between BF models and other theories. This problem has been considered in literature in relation with self-interactions in various classes of BF models [46, 47, 48, 49, 50, 51, 52] and couplings to matter fields [53] and vector fields [54, 55] by using the powerful BRST cohomological reformulation of the problem of constructing consistent interactions. Other aspects concerning interacting, topological BF models can be found in [56, 57, 58]. On the other hand, models with p-form

gauge fields play an important role in string and superstring theory as well as in supergravity. Based on these considerations, the study of interactions between BF models and two-forms appears as a topic that might enlighten certain aspects in both gravity and supergravity theories.

Our strategy goes as follows. Initially, we determine in Section 2 the antifield-BRST symmetry of the free model, which splits as the sum between the Koszul-Tate differential and the exterior derivative along the gauge orbits, $s = \delta + \gamma$. Then, in Section 3 we briefly present the reformulation of the problem of constructing consistent interactions in gauge field theories in terms of the deformation of the solution to the master equation. Next, in Section 4 we determine the consistent deformations of the solution to the master equation for the model under consideration. The first-order deformation belongs to the local cohomology $H^0(s|d)$, where d is the exterior spacetime derivative. The computation of the cohomological space $H^0(s|d)$ proceeds by expanding the co-cycles according to the antighost number and further using the cohomological groups $H(\gamma)$ and $H(\delta|d)$. We find that the first-order deformation is parameterized by 11 types of smooth functions of the undifferentiated scalar fields, which become restricted to fulfill 19 kinds of equations in order to produce a deformation that is consistent to all orders in the coupling constant. With the help of these equations we show that the remaining deformations, of orders 2 and higher, can be taken to vanish. The identification of the interacting model is developed in Section 5. All the interaction vertices are derivative-free. Among the cross-couplings between the collection of BF models and the set of two-form gauge fields we find a generalized version of non-Abelian Freedman-Townsend vertex. (By 'generalized' we mean that its form is identical with the standard non-Abelian Freedman-Townsend vertex up to the point that the structure constants of a Lie algebra are replaced here with some functions depending on the undifferentiated scalar fields from the BF sector.) Meanwhile, both the gauge transformations corresponding to the coupled model and their algebra are deformed with respect to the initial Abelian theory in such a way that the new gauge algebra becomes open and the reducibility relations only close on-shell (on the stationary surface of deformed field equations). It is interesting to mention that by contrast to the standard non-Abelian Freedman-Townsend model, where the auxiliary vector fields are gauge-invariant, here these fields gain nonvanishing gauge transformations, proportional with some BF gauge parameters. In the end of Section 5 we comment on several classes of solutions to the equations satisfied by the various functions of the scalar fields

that parameterize the deformed solution to the master equation. Section 6 closes the paper with the main conclusions. The present paper also contains 4 appendices, in which various notations used in the main body of the paper as well as some formulas concerning the gauge structure of the interacting model are listed.

2 Free model: Lagrangian formulation and BRST symmetry

The starting point is given by a free theory in four spacetime dimensions that describes a finite collection of BF models and a finite set of two-form gauge fields, with the Lagrangian action

$$S_0[A^a_\mu, H^a_\mu, \varphi_a, B^{\mu\nu}_a, V^A_{\mu\nu}, V^A_\mu] = \int d^4x \left(H^a_\mu \partial^\mu \varphi_a + \tfrac{1}{2} B^{\mu\nu}_a \partial_{[\mu} A^a_{\nu]} + \tfrac{1}{2} V^{\mu\nu}_A F^A_{\mu\nu} + \tfrac{1}{2} V^A_\mu V^\mu_A \right). \quad (2.1)$$

Each of the BF models from the collection (to be indexed by lower case letters a, b, etc.) comprises a scalar field φ_a, two kinds of one-forms A^a_μ and H^a_μ, and a two-form $B^{\mu\nu}_a$. The action for the set of Abelian two-forms decomposes as a sum of individual two-form actions, indexed via capital Latin letters (A, B, etc.). Each two-form action is written in first-order form as an Abelian Freedman-Townsend action, in terms of a two-form $V^{\mu\nu}_A$ and of an auxiliary vector V^A_μ, with the Abelian field strength $F^A_{\mu\nu} = \partial_{[\mu} V^A_{\nu]}$. The collection indices from the two-form sector are lowered with the (non-degenerate) metric k_{AB} induced by the Lagrangian density $\frac{1}{2}\left(V^{\mu\nu}_A F^A_{\mu\nu} + V^A_\mu V^\mu_A \right)$ from (2.1) (i.e. $F^{\mu\nu}_A = k_{AB} F^{B\mu\nu}$) and are raised with its inverse, of elements k^{AB}. Of course, we consider the general situation, where the two types of collection indexes run independently one from each other. Everywhere in this paper the notation $[\mu \ldots \nu]$ signifies complete antisymmetry with respect to the (Lorentz) indices between brackets, with the conventions that the minimum number of terms is always used and the result is never divided by the number of terms. Action (2.1) is found invariant under the gauge transformations

$$\delta_\epsilon A^a_\mu = \partial_\mu \epsilon^a, \quad \delta_\epsilon H^a_\mu = -2\partial^\nu \epsilon^a_{\nu\mu}, \quad \delta_\epsilon \varphi_a = 0, \quad (2.2)$$

$$\delta_\epsilon B^{\mu\nu}_a = -3\partial_\rho \epsilon^{\rho\mu\nu}_a, \quad \delta_\epsilon V^A_{\mu\nu} = \varepsilon_{\mu\nu\rho\lambda} \partial^\rho \epsilon^{A\lambda}, \quad \delta_\epsilon V^A_\mu = 0, \quad (2.3)$$

where all the gauge parameters are bosonic, with $\epsilon^a_{\mu\nu}$ and $\epsilon_a^{\mu\nu\rho}$ completely antisymmetric. It is easy to see that the above gauge transformations are Abelian and off-shell (everywhere in the space of field histories, not only on the stationary surface of field equations for (2.1)), second-order reducible. Indeed, related to the first-order reducibility, we observe that if we make the transformations $\epsilon^a_{\mu\nu}(\theta) = -3\partial^\lambda \theta^a_{\lambda\mu\nu}$, $\epsilon_a^{\mu\nu\rho}(\theta) = -4\partial_\lambda \theta_a^{\lambda\mu\nu\rho}$, $\epsilon^{A\lambda}(\theta) = \partial^\lambda \theta^A$, with θs arbitrary, bosonic functions, completely antisymmetric (where applicable) in their Lorentz indices, then the corresponding gauge transformations identically vanish, $\delta_{\epsilon(\theta)} H^a_\mu = 0$, $\delta_{\epsilon(\theta)} B_a^{\mu\nu} = 0$, $\delta_{\epsilon(\theta)} V^A_{\mu\nu} = 0$. The last two transformation laws of the gauge parameters can be further annihilated by trivial transformations only: $\epsilon_a^{\mu\nu\rho}(\theta) = 0$ if and only if $\theta_a^{\lambda\mu\nu\rho} = 0$ and $\epsilon^{A\lambda}(\theta) = 0$ if and only if $\theta^A = 0$, so there is no higher-order reducibility associated with them. By contrast, the first one can be made to vanish strongly via the transformation $\theta^a_{\lambda\mu\nu}(\omega) = -4\partial^\alpha \omega^a_{\alpha\lambda\mu\nu}$, with $\omega^a_{\alpha\lambda\mu\nu}$ an arbitrary, completely antisymmetric, bosonic function (which indeed produces $\epsilon^a_{\mu\nu}(\theta(\omega)) = 0$), but there is no nontrivial transformation of $\omega^a_{\alpha\lambda\mu\nu}$ such that $\theta^a_{\lambda\mu\nu}$ becomes zero. Thus, the reducibility of (2.2)–(2.3) stops at order 2 and holds off-shell.

In order to construct the BRST symmetry of this free theory, we introduce the field/ghost and antifield spectra

$$\Phi^{\alpha_0} = \left(A^a_\mu, H^a_\mu, \varphi_a, B_a^{\mu\nu}, V^A_{\mu\nu}, V^A_\mu\right), \tag{2.4}$$

$$\Phi^*_{\alpha_0} = \left(A_a^{*\mu}, H_a^{*\mu}, \varphi^{*a}, B^{*a}_{\mu\nu}, V_A^{*\mu\nu}, V_A^{*\mu}\right), \tag{2.5}$$

$$\eta^{\alpha_1} = \left(\eta^a, C^a_{\mu\nu}, \eta_a^{\mu\nu\rho}, C^A_\mu\right), \tag{2.6}$$

$$\eta^*_{\alpha_1} = \left(\eta^*_a, C_a^{*\mu\nu}, \eta^{*a}_{\mu\nu\rho}, C_A^{*\mu}\right), \tag{2.7}$$

$$\eta^{\alpha_2} = \left(C^a_{\mu\nu\rho}, \eta_a^{\mu\nu\rho\lambda}, C^A\right), \quad \eta^*_{\alpha_2} = \left(C_a^{*\mu\nu\rho}, \eta^{*a}_{\mu\nu\rho\lambda}, C^*_A\right), \tag{2.8}$$

$$\eta^{\alpha_3} = \left(C^a_{\mu\nu\rho\lambda}\right), \quad \eta^*_{\alpha_3} = \left(C_a^{*\mu\nu\rho\lambda}\right). \tag{2.9}$$

The fermionic ghosts η^{α_1} respectively correspond to the bosonic gauge parameters $\epsilon^{\alpha_1} = \left(\epsilon^a, \epsilon^a_{\mu\nu}, \epsilon_a^{\mu\nu\rho}, \epsilon^A_\mu\right)$, the bosonic ghosts for ghosts η^{α_2} are due to the first-order reducibility relations (the θ-parameters from the previous transformations), while the fermionic ghosts for ghosts for ghosts η^{α_3} are required by the second-order reducibility relations (the ω-function from the above). The star variables represent the antifields of the corresponding fields/ghosts. (Their Grassmann parities are respectively opposite to those of the associated fields/ghosts, in agreement with the general rules of the antifield-BRST method.)

Since both the gauge generators and the reducibility functions are field-

independent, it follows that the BRST differential reduces to

$$s = \delta + \gamma, \tag{2.10}$$

where δ is the Koszul-Tate differential and γ denotes the exterior longitudinal derivative. The Koszul-Tate differential is graded in terms of the antighost number (agh, $\mathrm{agh}\,(\delta) = -1$) and enforces a resolution of the algebra of smooth functions defined on the stationary surface of field equations for action (2.1), $C^\infty(\Sigma)$, $\Sigma : \delta S_0/\delta\Phi^{\alpha_0} = 0$. The exterior longitudinal derivative is graded in terms of the pure ghost number (pgh, $\mathrm{pgh}\,(\gamma) = 1$) and is correlated with the original gauge symmetry via its cohomology at pure ghost number 0 computed in $C^\infty(\Sigma)$, which is isomorphic to the algebra of physical observables for the free theory. These two degrees do not interfere ($\mathrm{agh}\,(\gamma) = 0$, $\mathrm{pgh}\,(\delta) = 0$). The pure ghost number and antighost number of BRST generators (2.4)–(2.9) are valued as follows:

$$\mathrm{pgh}\,(\Phi^{\alpha_0}) = 0, \quad \mathrm{pgh}\,(\eta^{\alpha_1}) = 1, \quad \mathrm{pgh}\,(\eta^{\alpha_2}) = 2, \quad \mathrm{pgh}\,(\eta^{\alpha_3}) = 3, \tag{2.11}$$

$$\mathrm{pgh}\left(\Phi^*_{\alpha_0}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_1}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_2}\right) = \mathrm{pgh}\left(\eta^*_{\alpha_3}\right) = 0, \tag{2.12}$$

$$\mathrm{agh}\,(\Phi^{\alpha_0}) = \mathrm{agh}\,(\eta^{\alpha_1}) = \mathrm{agh}\,(\eta^{\alpha_2}) = \mathrm{agh}\,(\eta^{\alpha_3}) = 0, \tag{2.13}$$

$$\mathrm{agh}\left(\Phi^*_{\alpha_0}\right) = 1, \quad \mathrm{agh}\left(\eta^*_{\alpha_1}\right) = 2, \quad \mathrm{agh}\left(\eta^*_{\alpha_2}\right) = 3, \quad \mathrm{agh}\left(\eta^*_{\alpha_3}\right) = 4, \tag{2.14}$$

where the actions of δ and γ on them read as

$$\delta\Phi^{\alpha_0} = \delta\eta^{\alpha_1} = \delta\eta^{\alpha_2} = \delta\eta^{\alpha_3} = 0, \tag{2.15}$$

$$\delta A^{*\mu}_a = -\partial_\nu B^{\mu\nu}_a, \quad \delta H^{*\mu}_a = -\partial^\mu \varphi_a, \quad \delta\varphi^{*a} = \partial^\mu H^a_\mu, \tag{2.16}$$

$$\delta B^{*a}_{\mu\nu} = -\tfrac{1}{2}\partial_{[\mu} A^a_{\nu]}, \quad \delta V^{*\mu\nu}_A = -\tfrac{1}{2} F^{\mu\nu}_A, \quad \delta V^{*\mu}_A = -\left(V^\mu_A + \partial_\nu V^{\mu\nu}_A\right), \tag{2.17}$$

$$\delta\eta^*_a = -\partial_\mu A^{*\mu}_a, \quad \delta C^{*\mu\nu}_a = \partial^{[\mu} H^{*\nu]}_a, \quad \delta\eta^{*a}_{\mu\nu\rho} = \partial_{[\mu} B^{*a}_{\nu\rho]}, \tag{2.18}$$

$$\delta C^{*\mu}_A = \varepsilon^{\mu\nu\rho\lambda}\partial_\nu V^*_{A\rho\lambda}, \quad \delta C^{*\mu\nu\rho}_a = -\partial^{[\mu} C^{*\nu\rho]}_a, \tag{2.19}$$

$$\delta\eta^{*a}_{\mu\nu\rho\lambda} = -\partial_{[\mu}\eta^{*a}_{\nu\rho\lambda]}, \quad \delta C^*_A = \partial_\mu C^{*\mu}_A, \quad \delta C^{*\mu\nu\rho\lambda}_a = \partial^{[\mu} C^{*\nu\rho\lambda]}_a, \tag{2.20}$$

$$\gamma\Phi^*_{\alpha_0} = \gamma\eta^*_{\alpha_1} = \gamma\eta^*_{\alpha_2} = \gamma\eta^*_{\alpha_3} = 0, \tag{2.21}$$

$$\gamma A^a_\mu = \partial_\mu \eta^a, \quad \gamma H^a_\mu = 2\partial^\nu C^a_{\mu\nu}, \quad \gamma B^{\mu\nu}_a = -3\partial_\rho \eta^{\mu\nu\rho}_a, \tag{2.22}$$

$$\gamma\varphi_a = 0 = \gamma V^A_\mu, \quad \gamma V^A_{\mu\nu} = \varepsilon_{\mu\nu\rho\lambda}\partial^\rho C^{A\lambda}, \quad \gamma\eta^a = 0, \tag{2.23}$$

$$\gamma C^a_{\mu\nu} = -3\partial^\rho C^a_{\mu\nu\rho}, \quad \gamma\eta^{\mu\nu\rho}_a = 4\partial_\lambda \eta^{\mu\nu\rho\lambda}_a, \quad \gamma C^A_\mu = \partial_\mu C^A, \tag{2.24}$$

$$\gamma C^a_{\mu\nu\rho} = 4\partial^\lambda C^a_{\mu\nu\rho\lambda}, \quad \gamma\eta^{\mu\nu\rho\lambda}_a = \gamma C^A = 0, \quad \gamma C^a_{\mu\nu\rho\lambda} = 0. \tag{2.25}$$

The overall degree of the BRST complex is named ghost number (gh) and is defined like the difference between the pure ghost number and the antighost

number, such that $\mathrm{gh}(\delta) = \mathrm{gh}(\gamma) = \mathrm{gh}(s) = 1$. The BRST symmetry admits a canonical action $s\cdot = \left(\cdot, \bar{S}\right)$ in an antibracket structure $(,)$, where its canonical generator is a bosonic functional of ghost number 0 ($\varepsilon\left(\bar{S}\right) = 0$, $\mathrm{gh}\left(\bar{S}\right) = 0$) that satisfies the classical master equation $\left(\bar{S}, \bar{S}\right) = 0$. In the case of the free theory under discussion, the solution to the master equation takes the form

$$\begin{aligned}\bar{S} = S_0 \quad + \quad & \int d^4x \left(A_a^{*\mu}\partial_\mu\eta^a + 2H_a^{*\mu}\partial^\nu C_{\mu\nu}^a - 3B_{\mu\nu}^{*a}\partial_\rho\eta_a^{\mu\nu\rho}\right. \\ & +\varepsilon_{\mu\nu\rho\lambda}V^{*A\mu\nu}\partial^\rho C_A^\lambda - 3C_a^{*\mu\nu}\partial^\rho C_{\mu\nu\rho}^a + 4\eta_{\mu\nu\rho}^{*a}\partial_\lambda\eta_a^{\mu\nu\rho\lambda} \\ & \left.+C_\mu^{*A}\partial^\mu C_A + 4C_a^{*\mu\nu\rho}\partial^\lambda C_{\mu\nu\rho\lambda}^a\right) \end{aligned} \tag{2.26}$$

and contains pieces of antighost number ranging from 0 to 3.

3 Deformation of the solution to the master equation: a brief review

We begin with a "free" gauge theory, described by a Lagrangian action $S_0^{\mathrm{L}}\left[\Phi^{\alpha_0}\right]$, invariant under some gauge transformations $\delta_\epsilon\Phi^{\alpha_0} = Z^{\alpha_0}_{\ \ \alpha_1}\epsilon^{\alpha_1}$, i.e. $\frac{\delta S_0^{\mathrm{L}}}{\delta\Phi^{\alpha_0}}Z^{\alpha_0}_{\ \ \alpha_1} = 0$, and consider the problem of constructing consistent interactions among the fields Φ^{α_0} such that the couplings preserve both the field spectrum and the original number of gauge symmetries. This matter is addressed by means of reformulating the problem of constructing consistent interactions as a deformation problem of the solution to the master equation corresponding to the "free" theory [20, 21]. Such a reformulation is possible due to the fact that the solution to the master equation contains all the information on the gauge structure of the theory. If an interacting gauge theory can be consistently constructed, then the solution $\bar{S}$ to the master equation $\left(\bar{S}, \bar{S}\right) = 0$ associated with the "free" theory can be deformed into a solution S

$$\bar{S} \rightarrow S = \bar{S} + \lambda S_1 + \lambda^2 S_2 + \cdots = \bar{S} + \lambda\int d^Dx\, a + \lambda^2\int d^Dx\, b + \cdots \tag{3.1}$$

of the master equation for the deformed theory

$$(S, S) = 0, \tag{3.2}$$

such that both the ghost and antifield spectra of the initial theory are preserved. Equation (3.2) splits, according to the various orders in the coupling constant (deformation parameter) λ, into a tower of equations:

$$\left(\bar{S},\bar{S}\right) = 0, \tag{3.3}$$
$$2\left(S_1,\bar{S}\right) = 0, \tag{3.4}$$
$$2\left(S_2,\bar{S}\right)+\left(S_1,S_1\right) = 0, \tag{3.5}$$
$$\left(S_3,\bar{S}\right)+\left(S_1,S_2\right) = 0, \tag{3.6}$$
$$\vdots$$

Equation (3.3) is fulfilled by hypothesis. The next equation requires that the first-order deformation of the solution to the master equation, S_1, is a cocycle of the "free" BRST differential, $sS_1 = 0$. However, only cohomologically nontrivial solutions to (3.4) should be taken into account, as the BRST-exact ones can be eliminated by some (in general nonlinear) field redefinitions. This means that S_1 pertains to the ghost number 0 cohomological space of s, $H^0(s)$, which is generically nonempty because it is isomorphic to the space of physical observables of the "free" theory. It has been shown (by the triviality of the antibracket map in the cohomology of the BRST differential) that there are no obstructions in finding solutions to the remaining equations, namely (3.5), (3.6), etc. However, the resulting interactions may be nonlocal, and obstructions might even appear if one insists on their locality. The analysis of these obstructions can be carried out by means of standard cohomological techniques.

4 Consistent interactions between a collection of topological BF models and a set of Abelian two-forms

This section is devoted to the investigation of consistent interactions that can be introduced between a collection of topological BF models and a set of Abelian two-forms in four spacetime dimensions. This matter is addressed in the context of the antifield-BRST deformation procedure briefly addressed in the above and relies on computing the solutions to equations (3.4)–(3.6), etc., with the help of the free BRST cohomology.

4.1 Standard material: basic cohomologies

For obvious reasons, we consider only smooth, local, Lorentz covariant, and Poincaré invariant deformations (i.e., we do not allow explicit dependence on the spacetime coordinates). Moreover, we require the preservation of the number of derivatives on each field with respect to the free theory (derivative-order assumption). The smoothness of the deformations refers to the fact that the deformed solution to the master equation, (3.1), is smooth in the coupling constant λ and reduces to the original solution, (2.26), in the free limit ($\lambda = 0$). The preservation of the number of derivatives on each field with respect to the free theory means here that the following two requirements must be simultaneously satisfied: (i) the derivative order of the equations of motion on each field is the same for the free and for the interacting theory, respectively; (ii) the maximum number of derivatives allowed within the interaction vertices is equal to 2, i.e. the maximum number of derivatives from the free Lagrangian. If we make the notation $S_1 = \int d^4x\, a$, with a a local function, then equation (3.4), which we have seen that controls the first-order deformation, takes the local form

$$sa = \partial_\mu m^\mu, \quad \mathrm{gh}\,(a) = 0, \quad \varepsilon\,(a) = 0, \tag{4.1}$$

for some local m^μ. It shows that the nonintegrated density of the first-order deformation pertains to the local cohomology of s in ghost number 0, $a \in H^0\,(s|d)$, where d denotes the exterior spacetime differential. The solution to (4.1) is unique up to s-exact pieces plus divergences

$$a \rightarrow a + sb + \partial_\mu n^\mu,\ \mathrm{gh}\,(b) = -1,\ \varepsilon\,(b) = 1,\ \mathrm{gh}\,(n^\mu) = 0,\ \varepsilon\,(n^\mu) = 0. \tag{4.2}$$

At the same time, if the general solution to (4.1) is found to be completely trivial, $a = sb + \partial_\mu n^\mu$, then it can be made to vanish $a = 0$.

In order to analyze equation (4.1) we develop a according to the antighost number

$$a = \sum_{i=0}^{I} a_i, \quad \mathrm{agh}\,(a_i) = i, \quad \mathrm{gh}\,(a_i) = 0, \quad \varepsilon\,(a_i) = 0, \tag{4.3}$$

and assume, without loss of generality, that the above decomposition stops at some finite value of I. This can be shown, for instance, like in [59] (Section 3), under the sole assumption that the interacting Lagrangian at the first order

in the coupling constant, a_0, has a finite, but otherwise arbitrary derivative order. Inserting decomposition (4.3) into equation (4.1) and projecting it on the various values of the antighost number, we obtain the tower of equations

$$\gamma a_I = \partial_\mu \overset{(I)^\mu}{m}, \tag{4.4}$$

$$\delta a_I + \gamma a_{I-1} = \partial_\mu \overset{(I-1)^\mu}{m}, \tag{4.5}$$

$$\delta a_i + \gamma a_{i-1} = \partial_\mu \overset{(i-1)^\mu}{m}, \quad 1 \leq i \leq I-1, \tag{4.6}$$

where $\left(\overset{(i)^\mu}{m}\right)_{i=\overline{0,I}}$ are some local currents with agh $\left(\overset{(i)^\mu}{m}\right) = i$. Equation (4.4) can be replaced in strictly positive values of the antighost number by

$$\gamma a_I = 0, \quad I > 0. \tag{4.7}$$

Due to the second-order nilpotency of γ ($\gamma^2 = 0$), the solution to (4.7) is clearly unique up to γ-exact contributions

$$a_I \rightarrow a_I + \gamma b_I, \quad \mathrm{agh}\,(b_I) = I, \quad \mathrm{pgh}\,(b_I) = I-1, \quad \varepsilon\,(b_I) = 1. \tag{4.8}$$

Meanwhile, if it turns out that a_I exclusively reduces to γ-exact terms, $a_I = \gamma b_I$, then it can be made to vanish, $a_I = 0$. In other words, the nontriviality of the first-order deformation a is translated at its highest antighost number component into the requirement that $a_I \in H^I(\gamma)$, where $H^I(\gamma)$ denotes the cohomology of the exterior longitudinal derivative γ in pure ghost number equal to I. So, in order to solve equation (4.1) (equivalent with (4.7) and (4.5)–(4.6)), we need to compute the cohomology of γ, $H(\gamma)$, and, as it will be made clear below, also the local homology of δ, $H(\delta|d)$.

On behalf of definitions (2.21)–(2.25) it is simple to see that $H(\gamma)$ is spanned by

$$F_{\bar{A}} = \left(\varphi_a, \partial_{[\mu} A^a_{\nu]}, \partial^\mu H^a_\mu, \partial_\mu B^{\mu\nu}_a, V^A_\mu, \tilde{F}^A_{\mu\nu\rho}\right), \tag{4.9}$$

the antifields

$$\chi^*_\Delta = \left(\Phi^*_{\alpha_0}, \eta^*_{\alpha_1}, \eta^*_{\alpha_2}, \eta^*_{\alpha_3}\right), \tag{4.10}$$

all of their spacetime derivatives as well as by the undifferentiated ghosts

$$\eta^{\bar{\Upsilon}} = \left(\eta^a, C^A, \eta^{\mu\nu\rho\lambda}_a, C^a_{\mu\nu\rho\lambda}\right). \tag{4.11}$$

In formula (4.9) we used the notation

$$\tilde{F}^A_{\mu\nu\rho} = \partial_{[\mu} \tilde{V}^A_{\nu\rho]}, \quad \tilde{V}^A_{\mu\nu} \equiv \tfrac{1}{2}\varepsilon_{\mu\nu\rho\lambda} V^{A\rho\lambda}. \tag{4.12}$$

(The derivatives of the ghosts $\eta^{\bar{\Upsilon}}$ are removed from $H(\gamma)$ since they are γ-exact, in agreement with the first relation from (2.22), the last formula in (2.24), the second equation in (2.24), and the first definition from (2.25).) If we denote by $e^{M}\left(\eta^{\bar{\Upsilon}}\right)$ the elements with pure ghost number M of a basis in the space of the polynomials in the ghosts (4.11), then it follows that the general solution to equation (4.7) takes the form

$$a_{I}=\alpha_{I}\left(\left[F_{\bar{A}}\right],\left[\chi_{\Delta}^{*}\right]\right)e^{I}\left(\eta^{\bar{\Upsilon}}\right), \tag{4.13}$$

where $\mathrm{agh}\left(\alpha_{I}\right)=I$ and $\mathrm{pgh}\left(e^{I}\right)=I$. The notation $f([q])$ means that f depends on q and its spacetime derivatives up to a finite order. The objects α_{I} (obviously nontrivial in $H^{0}(\gamma)$) will be called "invariant polynomials". The result that we can replace equation (4.4) with the less obvious one (4.7) is a nice consequence of the fact that the cohomology of the exterior spacetime differential is trivial in the space of invariant polynomials in strictly positive antighost numbers.

Inserting (4.13) in (4.5) we obtain that a necessary (but not sufficient) condition for the existence of (nontrivial) solutions a_{I-1} is that the invariant polynomials α_{I} are (nontrivial) objects from the local cohomology of Koszul-Tate differential $H(\delta|d)$ in antighost number $I>0$ and in pure ghost number 0,

$$\delta\alpha_{I}=\partial_{\mu}\stackrel{(I-1)^{\mu}}{j},\quad \mathrm{agh}\left(\stackrel{(I-1)^{\mu}}{j}\right)=I-1,\quad \mathrm{pgh}\left(\stackrel{(I-1)^{\mu}}{j}\right)=0. \tag{4.14}$$

We recall that the local cohomology $H(\delta|d)$ is completely trivial in both strictly positive antighost *and* pure ghost numbers (for instance, see [60], Theorem 5.4, and [61]), so from now on it is understood that by $H(\delta|d)$ we mean the local cohomology of δ at pure ghost number 0. Using the fact that the free BF model under study is a linear gauge theory of Cauchy order equal to 4 and the general result from [60, 61], according to which the local cohomology of the Koszul-Tate differential is trivial in antighost numbers strictly greater than its Cauchy order, we can state that

$$H_{J}(\delta|d)=0\quad \text{for all}\quad J>4, \tag{4.15}$$

where $H_{J}(\delta|d)$ represents the local cohomology of the Koszul-Tate differential in antighost number J. Moreover, if the invariant polynomial α_{J}, with

$\mathrm{agh}(\alpha_J) = J \geq 4$, is trivial in $H_J(\delta|d)$, then it can be taken to be trivial also in $H_J^{\mathrm{inv}}(\delta|d)$

$$\left(\alpha_J = \delta b_{J+1} + \partial_\mu \overset{(J)^\mu}{c}, \mathrm{agh}(\alpha_J) = J \geq 4\right) \Rightarrow \alpha_J = \delta \beta_{J+1} + \partial_\mu \overset{(J)^\mu}{\gamma}, \quad (4.16)$$

with both β_{J+1} and $\overset{(J)^\mu}{\gamma}$ invariant polynomials. Here, $H_J^{\mathrm{inv}}(\delta|d)$ denotes the invariant characteristic cohomology in antighost number J (the local cohomology of the Koszul-Tate differential in the space of invariant polynomials). (An element of $H_I^{\mathrm{inv}}(\delta|d)$ is defined via an equation like (4.14), but with the corresponding current an invariant polynomial.). This result together with (4.15) ensures that the entire invariant characteristic cohomology in antighost numbers strictly greater than 4 is trivial

$$H_J^{\mathrm{inv}}(\delta|d) = 0 \quad \text{for all} \quad J > 4. \quad (4.17)$$

The nontrivial representatives of $H_J(\delta|d)$ and of $H_J^{\mathrm{inv}}(\delta|d)$ for $J \geq 2$ depend neither on $\left(\partial_{[\mu} A^a_{\nu]}, \partial^\mu H^a_\mu, \partial_\mu B^{\mu\nu}_a, \tilde{F}^A_{\mu\nu\rho}\right)$ nor on the spacetime derivatives of $F_{\bar{A}}$ defined in (4.9), but only on the undifferentiated scalar fields and auxiliary vector fields from the two-form sector, $\left(\varphi_a, V^A_\mu\right)$. With the help of relations (2.15)–(2.20), it can be shown that $H_4^{\mathrm{inv}}(\delta|d)$ is generated by the elements

$$\begin{aligned}
(P_\Lambda(W))^{\mu\nu\rho\lambda} &= \frac{\partial W_\Lambda}{\partial \varphi_a} C^{*\mu\nu\rho\lambda}_a + \frac{\partial^2 W_\Lambda}{\partial \varphi_a \partial \varphi_b}\left(H^{*[\mu}_a C^{*\nu\rho\lambda]}_b + C^{*[\mu\nu}_a C^{*\rho\lambda]}_b\right) \\
&\quad + \frac{\partial^3 W_\Lambda}{\partial \varphi_a \partial \varphi_b \partial \varphi_c} H^{*[\mu}_a H^{*\nu}_b C^{*\rho\lambda]}_c \\
&\quad + \frac{\partial^4 W_\Lambda}{\partial \varphi_a \partial \varphi_b \partial \varphi_c \partial \varphi_d} H^{*\mu}_a H^{*\nu}_b H^{*\rho}_c H^{*\lambda}_d, \quad (4.18)
\end{aligned}$$

where $W_\Lambda = W_\Lambda(\varphi_a)$ are arbitrary, smooth functions depending only on the undifferentiated scalar fields φ_a and Λ is some multi-index (composed of internal and/or Lorentz indices). Indeed, direct computation yields

$$\delta(P_\Lambda(W))^{\mu\nu\rho\lambda} = \partial^{[\mu}(P_\Lambda(W))^{\nu\rho\lambda]}, \quad \mathrm{agh}\left((P_\Lambda(W))^{\nu\rho\lambda}\right) = 3, \quad (4.19)$$

where we made the notation

$$(P_\Lambda(W))^{\mu\nu\rho} = \frac{\partial W_\Lambda}{\partial \varphi_a} C^{*\mu\nu\rho}_a + \frac{\partial^2 W_\Lambda}{\partial \varphi_a \partial \varphi_b} H^{*[\mu}_a C^{*\nu\rho]}_b$$

$$+\frac{\partial^3 W_\Lambda}{\partial\varphi_a\partial\varphi_b\partial\varphi_c} H_a^{*\mu} H_b^{*\nu} H_c^{*\rho}. \tag{4.20}$$

It is clear that $(P_\Lambda(W))^{\mu\nu\rho}$ is an invariant polynomial. By applying the operator δ on it, we have that

$$\delta (P_\Lambda(W))^{\mu\nu\rho} = -\partial^{[\mu} (P_\Lambda(W))^{\nu\rho]}, \quad \mathrm{agh}((P_\Lambda(W))^{\nu\rho}) = 2, \tag{4.21}$$

where we employed the convention

$$(P_\Lambda(W))^{\mu\nu} = \frac{\partial W_\Lambda}{\partial\varphi_a} C_a^{*\mu\nu} + \frac{\partial^2 W_\Lambda}{\partial\varphi_a\partial\varphi_b} H_a^{*\mu} H_b^{*\nu}. \tag{4.22}$$

Since $(P_\Lambda(W))^{\mu\nu}$ is also an invariant polynomial, from (4.21) it follows that $(P_\Lambda(W))^{\mu\nu\rho}$ belongs to $H_3^{\mathrm{inv}}(\delta|d)$. Moreover, further calculations produce

$$\delta (P_\Lambda(W))^{\mu\nu} = \partial^{[\mu} (P_\Lambda(W))^{\nu]}, \quad \mathrm{agh}((P_\Lambda(W))^{\nu}) = 1, \tag{4.23}$$

with

$$(P_\Lambda(W))^{\mu} = \frac{\partial W_\Lambda}{\partial\varphi_a} H_a^{*\mu}. \tag{4.24}$$

Due to the fact that $(P_\Lambda(W))^{\mu}$ is an invariant polynomial, we deduce that $(P_\Lambda(W))^{\mu\nu}$ pertains to $H_2^{\mathrm{inv}}(\delta|d)$. Using again the actions of δ on the BRST generators, it can be proved that $H_3^{\mathrm{inv}}(\delta|d)$ is spanned, beside the elements $(P_\Lambda(W))^{\mu\nu\rho}$ given in (4.20), also by the objects

$$\begin{aligned} Q_\Lambda(f) \;=\;& f_\Lambda^A C_A^* - \left(P_\Lambda^A(f)\right)^\mu C_{A\mu}^* - \tfrac{1}{2}\varepsilon_{\mu\nu\rho\lambda}\left(\tfrac{1}{3}\left(P_\Lambda^A(f)\right)^{\mu\nu\rho} V_A^\lambda \right. \\ & \left. + \left(P_\Lambda^A(f)\right)^{\mu\nu} V_A^{*\rho\lambda}\right) \end{aligned} \tag{4.25}$$

and by the undifferentiated antifields $\eta_{\mu\nu\rho\lambda}^{*a}$ (according to the first definition from (2.20)). In formula (4.25) $f_\Lambda^A = f_\Lambda^A(\varphi_a)$ are some arbitrary, smooth functions of the undifferentiated scalar fields φ_a carrying at least an internal index A from the two-form sector and possibly a supplementary multi-index Λ. The factors $\left(P_\Lambda^A(f)\right)^\mu$, $\left(P_\Lambda^A(f)\right)^{\mu\nu}$, and $\left(P_\Lambda^A(f)\right)^{\mu\nu\rho}$ read as in (4.24), (4.22), and (4.20), respectively, with $W_\Lambda(\varphi_a) \to f_\Lambda^A(\varphi_a)$. Concerning $Q_\Lambda(f)$, we have that

$$\delta Q_\Lambda(f) = \partial_\mu (Q_\Lambda(f))^\mu, \quad \mathrm{agh}((Q_\Lambda(f))^\mu) = 2, \tag{4.26}$$

where we employed the notation

$$(Q_\Lambda(f))^\mu = f_\Lambda^A C_A^{*\mu} + \varepsilon^{\mu\nu\rho\lambda}\left(\left(P_\Lambda^A(f)\right)_\nu V_{A\rho\lambda}^* + \tfrac{1}{2}\left(P_\Lambda^A(f)\right)_{\nu\rho} V_{A\lambda}\right). \quad (4.27)$$

With the help of definitions (2.15)–(2.20) it can be checked that

$$\delta(Q_\Lambda(f))^\mu = \partial_\nu(Q_\Lambda(f))^{\mu\nu}, \quad \mathrm{agh}\left((Q_\Lambda(f))^{\mu\nu}\right) = 1, \quad (4.28)$$

where we made the notation

$$(Q_\Lambda(f))^{\mu\nu} = \varepsilon^{\mu\nu\rho\lambda}\left(f_\Lambda^A V_{A\rho\lambda}^* + \left(P_\Lambda^A(f)\right)_\rho V_{A\lambda}\right). \quad (4.29)$$

Direct computation shows that the objects

$$\begin{aligned} R_\Lambda(g) \;&=\; g_\Lambda^{AB}\left(C_A^{*\mu} V_{B\mu} + \tfrac{1}{2}\varepsilon_{\mu\nu\rho\lambda} V_A^{*\mu\nu} V_B^{*\rho\lambda}\right) \\ &\quad -\varepsilon_{\mu\nu\rho\lambda}\left(\left(P_\Lambda^{AB}(g)\right)^\mu V_A^{*\nu\rho} + \tfrac{1}{4}\left(P_\Lambda^{AB}(g)\right)^{\mu\nu} V_A^\rho\right) V_B^\lambda \end{aligned} \quad (4.30)$$

satisfy

$$\delta R_\Lambda(g) = \partial^\mu\left(R_\Lambda(g)\right)_\mu, \quad \mathrm{agh}\left(\left(R_\Lambda(g)\right)_\mu\right) = 1, \quad (4.31)$$

with

$$\left(R_\Lambda(g)\right)_\mu = -\varepsilon_{\mu\nu\rho\lambda}\left(g_\Lambda^{AB} V_A^{*\nu\rho} + \tfrac{1}{2}\left(P_\Lambda^{AB}(g)\right)^\nu V_A^\rho\right) V_B^\lambda. \quad (4.32)$$

In formulas (4.30) and (4.32) $g_\Lambda^{AB} = g_\Lambda^{AB}(\varphi_a)$ stand for some smooth functions of the undifferentiated scalar fields that in addition are antisymmetric with respect to A and B

$$g_\Lambda^{AB} = -g_\Lambda^{BA}. \quad (4.33)$$

Looking at their expressions, it is easy to see that all the quantities denoted by Qs or Rs are invariant polynomials. Putting together the above results we can state that $H_2^{\mathrm{inv}}(\delta|d)$ is spanned by $(P_\Lambda(W))^{\mu\nu}$ listed in (4.22), $(Q_\Lambda(f))^\mu$ expressed by (4.27), $R_\Lambda(g)$ given in (4.30), and the undifferentiated antifields $\eta_{\mu\nu\rho}^{*a}$ and η_a^* (in agreement with the last formula from (2.18) and the first definition in (2.18)).

In contrast to the spaces $(H_J(\delta|d))_{J\geq 2}$ and $\left(H_J^{\mathrm{inv}}(\delta|d)\right)_{J\geq 2}$, which are finite-dimensional, the cohomology $H_1(\delta|d)$ (known to be related to global symmetries and ordinary conservation laws) is infinite-dimensional since the theory is free. Fortunately, it will not be needed in the sequel.

The previous results on $H(\delta|d)$ and $H^{\mathrm{inv}}(\delta|d)$ in strictly positive antighost numbers are important because they control the obstructions to removing the antifields from the first-order deformation. More precisely, we can successively eliminate all the pieces of antighost number strictly greater that 4 from the nonintegrated density of the first-order deformation by adding solely trivial terms, so we can take, without loss of nontrivial objects, the condition $I \leq 4$ into (4.3). In addition, the last representative is of the form (4.13), where the invariant polynomial is necessarily a nontrivial object from $H_4^{\mathrm{inv}}(\delta|d)$.

4.2 First-order deformation

In the case $I = 4$ the nonintegrated density of the first-order deformation (see (4.3)) becomes

$$a = a_0 + a_1 + a_2 + a_3 + a_4. \tag{4.34}$$

We can further decompose a in a natural manner as a sum between two kinds of deformations

$$a = a^{(\mathrm{BF})} + a^{(\mathrm{int})}, \tag{4.35}$$

where $a^{(\mathrm{BF})}$ contains only fields/ghosts/antifields from the BF sector and $a^{(\mathrm{int})}$ describes the cross-interactions between the two theories. Strictly speaking, we should have added to (4.35) also a component $a^{(\mathrm{V})}$ that involves only the two-form field sector. As it will be seen at the end of this subsection, $a^{(\mathrm{V})}$ will be automatically included into $a^{(\mathrm{int})}$. The piece $a^{(\mathrm{BF})}$ is completely known (see [50, 53, 52]) and (separately) satisfies an equation of the type (4.1). It admits a decomposition similar to (4.34)

$$a^{(\mathrm{BF})} = a_0^{(\mathrm{BF})} + a_1^{(\mathrm{BF})} + a_2^{(\mathrm{BF})} + a_3^{(\mathrm{BF})} + a_4^{(\mathrm{BF})}, \tag{4.36}$$

where

$$\begin{aligned} a_4^{(\mathrm{BF})} = {} & (P_{ab}(W))^{\mu\nu\rho\lambda} \eta^a C^b_{\mu\nu\rho\lambda} - \tfrac{1}{4} (P^c_{ab}(M))_{\mu\nu\rho\lambda} \eta^a \eta^b \eta_c^{\mu\nu\rho\lambda} \\ & + \tfrac{1}{2} \varepsilon_{\mu\nu\rho\lambda} \Big(\left(P^{ab}(M)\right)^{\mu\nu\rho\lambda} \eta_{a\alpha\beta\gamma\delta} \eta_b^{\alpha\beta\gamma\delta} \\ & - \tfrac{1}{2\cdot(4!)^2} (P_{abcd}(M))^{\mu\nu\rho\lambda} \eta^a \eta^b \eta^c \eta^d \Big), \end{aligned} \tag{4.37}$$

$$a_3^{(\mathrm{BF})} = (P_{ab}(W))^{\mu\nu\rho} \left(-\eta^a C^b_{\mu\nu\rho} + 4 A^{a\lambda} C^b_{\mu\nu\rho\lambda}\right)$$

$$
\begin{aligned}
&+2\left(6\left(P_{ab}\left(W\right)\right)^{\mu\nu}B^{*a\rho\lambda}+4\left(P_{ab}\left(W\right)\right)^{\mu}\eta^{*a\nu\rho\lambda}+W_{ab}\eta^{*a\mu\nu\rho\lambda}\right)C^{b}_{\mu\nu\rho\lambda}\\
&+\tfrac{1}{2}\left(P^{c}_{ab}\left(M\right)\right)_{\mu\nu\rho}\left(\tfrac{1}{2}\eta^{a}\eta^{b}\eta^{\mu\nu\rho}_{c}-4A^{a}_{\lambda}\eta^{b}\eta^{\mu\nu\rho\lambda}_{c}\right)\\
&-\left(6\left(P^{c}_{ab}\left(M\right)\right)_{\mu\nu}B^{*a}_{\rho\lambda}+4\left(P^{c}_{ab}\left(M\right)\right)_{\mu}\eta^{*a}_{\nu\rho\lambda}+M^{c}_{ab}\eta^{*a}_{\mu\nu\rho\lambda}\right)\eta^{b}\eta^{\mu\nu\rho\lambda}_{c}\\
&-\varepsilon_{\mu\nu\rho\lambda}\left(P^{ab}\left(M\right)\right)_{\alpha\beta\gamma}\eta^{\alpha\beta\gamma}_{a}\eta^{\mu\nu\rho\lambda}_{b}-\tfrac{1}{3!\cdot 4!}\varepsilon^{\mu\nu\rho\lambda}\left(\left(P_{abcd}\left(M\right)\right)_{\mu\nu\rho}A^{a}_{\lambda}\right.\\
&+3\left(P_{abcd}\left(M\right)\right)_{\mu\nu}B^{*a}_{\rho\lambda}+2\left(P_{abcd}\left(M\right)\right)_{\mu}\eta^{*a}_{\nu\rho\lambda}\\
&\left.+M_{abcd}\eta^{*a}_{\mu\nu\rho\lambda}\right)\eta^{b}\eta^{c}\eta^{d},
\end{aligned}
\tag{4.38}
$$

$$
\begin{aligned}
a^{(\mathrm{BF})}_{2}=&\left(P_{ab}\left(W\right)\right)^{\mu\nu}\left(\eta^{a}C^{b}_{\mu\nu}-3A^{a\rho}C^{b}_{\mu\nu\rho}\right)-2\left(3\left(P_{ab}\left(W\right)\right)^{\mu}B^{*a\nu\rho}\right.\\
&\left.+W_{ab}\eta^{*a\mu\nu\rho}\right)C^{b}_{\mu\nu\rho}-\tfrac{1}{2}\left(P^{c}_{ab}\left(M\right)\right)^{\mu\nu}\left(\tfrac{1}{2}\eta^{a}\eta^{b}B_{c\mu\nu}-3A^{a\rho}\eta^{b}\eta_{c\mu\nu\rho}\right)\\
&+\left(3\left(P^{c}_{ab}\left(M\right)\right)_{\mu}B^{*a}_{\nu\rho}+M^{c}_{ab}\eta^{*a}_{\mu\nu\rho}\right)\eta^{b}\eta^{\mu\nu\rho}_{c}+\tfrac{1}{2}\left(-\left(P^{c}_{ab}\left(M\right)\right)_{\mu}A^{*\mu}_{c}\right.\\
&\left.+M^{c}_{ab}\eta^{*}_{c}\right)\eta^{a}\eta^{b}+\left(3\left(P^{c}_{ab}\left(M\right)\right)_{\mu\nu}A^{a}_{\rho}+12\left(P^{c}_{ab}\left(M\right)\right)_{\mu}B^{*a}_{\nu\rho}\right.\\
&\left.+4M^{c}_{ab}\eta^{*a}_{\mu\nu\rho}\right)A^{b}_{\lambda}\eta^{\mu\nu\rho\lambda}_{c}+\tfrac{9}{2}\varepsilon^{\mu\nu\rho\lambda}\left(P^{ab}\left(M\right)\right)_{\mu\nu}\eta_{a\rho\alpha\beta}\eta_{b\lambda}{}^{\alpha\beta}\\
&-6M^{c}_{ab}B^{*a}_{\mu\nu}B^{*b}_{\rho\lambda}\eta^{\mu\nu\rho\lambda}_{c}+\tfrac{1}{4\cdot 4!}\varepsilon^{\mu\nu\rho\lambda}\left(3\left(P_{abcd}\left(M\right)\right)_{\mu\nu}A^{a}_{\rho}A^{b}_{\lambda}\right.\\
&\left.+12\left(P_{abcd}\left(M\right)\right)_{\mu}B^{*a}_{\nu\rho}A^{b}_{\lambda}+4M_{abcd}\eta^{*a}_{\mu\nu\rho}A^{b}_{\lambda}-6M_{abcd}B^{*a}_{\mu\nu}B^{*b}_{\rho\lambda}\right)\eta^{c}\eta^{d}\\
&+\varepsilon_{\mu\nu\rho\lambda}\left(2\left(P^{ab}\left(M\right)\right)_{\alpha}A^{*\alpha}_{a}-2M^{ab}\eta^{*}_{a}\right.\\
&\left.+\left(P^{ab}\left(M\right)\right)_{\alpha\beta}B^{\alpha\beta}_{a}\right)\eta^{\mu\nu\rho\lambda}_{b},
\end{aligned}
\tag{4.39}
$$

$$
\begin{aligned}
a^{(\mathrm{BF})}_{1}=&\left(P_{ab}\left(W\right)\right)^{\mu}\left(-\eta^{a}H^{b}_{\mu}+2A^{a\nu}C^{b}_{\mu\nu}\right)+W_{ab}\left(2B^{*a}_{\mu\nu}C^{b\mu\nu}-\varphi^{*a}\eta^{b}\right)\\
&-\left(P^{c}_{ab}\left(M\right)\right)_{\mu}A^{a}_{\nu}\left(\eta^{b}B^{\mu\nu}_{c}+\tfrac{3}{2}A^{b}_{\rho}\eta^{\mu\nu\rho}_{c}\right)-M^{c}_{ab}\left(B^{*a}_{\mu\nu}\eta^{b}B^{\mu\nu}_{c}\right.\\
&\left.+A^{a}_{\mu}\eta^{b}A^{*\mu}_{c}+3B^{*a}_{\mu\nu}A^{b}_{\rho}\eta^{\mu\nu\rho}_{c}\right)\\
&+2\varepsilon_{\nu\rho\sigma\lambda}\left(\left(P^{ab}\left(M\right)\right)_{\mu}B^{\mu\nu}_{a}-M^{ab}A^{*\nu}_{a}\right)\eta^{\rho\sigma\lambda}_{b}\\
&+\tfrac{1}{4!}\varepsilon^{\mu\nu\rho\lambda}\left(\left(P_{abcd}\left(M\right)\right)_{\mu}A^{a}_{\nu}+3M_{abcd}B^{*a}_{\mu\nu}\right)A^{b}_{\rho}A^{c}_{\lambda}\eta^{d},
\end{aligned}
\tag{4.40}
$$

$$
\begin{aligned}
a^{(\mathrm{BF})}_{0}=&-W_{ab}A^{a\mu}H^{b}_{\mu}+\tfrac{1}{2}M^{c}_{ab}A^{a}_{\mu}A^{b}_{\nu}B^{\mu\nu}_{c}\\
&+\tfrac{1}{2}\varepsilon^{\mu\nu\rho\lambda}\left(M^{ab}B_{a\mu\nu}B_{b\rho\lambda}-\tfrac{1}{2\cdot 4!}M_{abcd}A^{a}_{\mu}A^{b}_{\nu}A^{c}_{\rho}A^{d}_{\lambda}\right).
\end{aligned}
\tag{4.41}
$$

In (4.37)–(4.41) the quantities denoted by $(P_{ab}(W))^{\mu_1\cdots\mu_k}$, $(P^c_{ab}(M))^{\mu_1\cdots\mu_k}$, $\left(P^{ab}(M)\right)^{\mu_1\cdots\mu_k}$, and $(P_{abcd}(M))^{\mu_1\cdots\mu_k}$ read as in (4.18), (4.20), (4.22), and (4.24) for $k=4$, $k=3$, $k=2$, and $k=1$, respectively, modulo the successive replacement of $W_\Lambda(\varphi_a)$ with the functions W_{ab}, M^c_{ab}, M^{ab}, and M_{abcd}, respectively. The last four kinds of functions depend only on the undifferentiated scalar fields and satisfy various symmetry/antisymmetry properties: M^c_{ab} are antisymmetric in their lower indices, M^{ab} are symmetric, and M_{abcd} are completely antisymmetric.

Due to the fact that $a^{(\mathrm{BF})}$ and $a^{(\mathrm{int})}$ involve different types of fields and $a^{(\mathrm{BF})}$ separately satisfies an equation of the type (4.1), it follows that $a^{(\mathrm{int})}$ is subject to the equation

$$sa^{(\mathrm{int})}=\partial_\mu m^{(\mathrm{int})\mu}, \tag{4.42}$$

for some local current $m^{(\mathrm{int})\mu}$. In the sequel we determine the general solution to (4.42) that complies with all the hypotheses mentioned in the beginning of the previous subsection.

In agreement with (4.34), the solution to the equation $sa^{(\mathrm{int})}=\partial_\mu m^{(\mathrm{int})\mu}$ can be decomposed as

$$a^{(\mathrm{int})}=a_0^{(\mathrm{int})}+a_1^{(\mathrm{int})}+a_2^{(\mathrm{int})}+a_3^{(\mathrm{int})}+a_4^{(\mathrm{int})}, \tag{4.43}$$

where the components on the right-hand side of (4.43) are subject to the equations

$$\gamma a_4^{(\mathrm{int})} = 0, \tag{4.44}$$

$$\delta a_k^{(\mathrm{int})}+\gamma a_{k-1}^{(\mathrm{int})} = \partial_\mu \overset{(k-1)}{m}{}^{(\mathrm{int})\mu}, \quad k=\overline{1,4}. \tag{4.45}$$

The piece $a_4^{(\mathrm{int})}$ as solution to equation (4.44) has the general form expressed by (4.13) for $I=4$, with α_4 from $H_4^{\mathrm{inv}}(\delta|d)$ and e^4 spanned by

$$\left(\eta^a\eta^b\eta^c\eta^d,\eta^a\eta^b\eta_c^{\mu\nu\rho\lambda},\eta^a C^b_{\mu\nu\rho\lambda},\eta_a^{\mu\nu\rho\lambda}\eta_b^{\alpha\beta\gamma\delta},\eta^a\eta^b C^A,C^A C^B,C^A\eta_a^{\mu\nu\rho\lambda}\right). \tag{4.46}$$

Taking into account the result that the general representative of $H_4^{\mathrm{inv}}(\delta|d)$ is given by (4.18) and recalling that $a_4^{(\mathrm{int})}$ should mix the BF and the two-form sectors (in order to provide cross-couplings), it follows that the eligible representatives of e^4 from (4.46) allowed to enter $a_4^{(\mathrm{int})}$ are those elements containing at least one ghost of the type C^A. Therefore, up to trivial, γ-exact terms, we can write

$$a_4^{(\mathrm{int})} = \tfrac{1}{2\cdot 4!}\varepsilon_{\mu\nu\rho\lambda}\left(\left(P_{abA}(N)\right)^{\mu\nu\rho\lambda}\eta^a\eta^b C^A+\left(P_{AB}(N)\right)^{\mu\nu\rho\lambda}C^A C^B\right)$$

$$+\left(P_A^a(N)\right)_{\mu\nu\rho\lambda} C^A \eta_a^{\mu\nu\rho\lambda}, \tag{4.47}$$

where the objects denoted by $\left(P_{abA}(N)\right)^{\mu\nu\rho\lambda}$, $\left(P_{AB}(N)\right)^{\mu\nu\rho\lambda}$, and respectively $\left(P_A^a(N)\right)_{\mu\nu\rho\lambda}$ are expressed as in (4.18), being generated by the arbitrary, smooth functions of the undifferentiated scalar fields $N_{abA}(\varphi_m)$, $N_{AB}(\varphi_m)$, and $N_A^a(\varphi_m)$, respectively. In addition, the functions $N_{abA}(\varphi_m)$ and $N_{AB}(\varphi_m)$ satisfy the symmetry/antisymmetry properties

$$N_{abA}(\varphi_m) = -N_{baA}(\varphi_m), \; N_{AB}(\varphi_m) = N_{BA}(\varphi_m). \tag{4.48}$$

Inserting (4.47) into equation (4.45) for $k = 4$ and using definitions (2.15)–(2.25), after some computation we obtain the interacting piece of antighost number 3 from the first-order deformation in the form

$$\begin{aligned} a_3^{(\mathrm{int})} = & -\left(P_A^a(N)\right)_{\mu\nu\rho}\left(C^A \eta_a^{\mu\nu\rho} + 4C_\lambda^A \eta_a^{\mu\nu\rho\lambda}\right) \\ & -\tfrac{1}{3!}\varepsilon^{\mu\nu\rho\lambda}\Big[\left(P_{abA}(N)\right)_{\mu\nu\rho}\eta^a\left(A_\lambda^b C^A + \tfrac{1}{2}\eta^b C_\lambda^A\right) \\ & +\left(P_{AB}(N)\right)_{\mu\nu\rho} C^A C_\lambda^B - \Big(3\left(P_{abA}(N)\right)_{\mu\nu} B_{\rho\lambda}^{*a} \\ & +2\left(P_{abA}(N)\right)_\mu \eta_{\nu\rho\lambda}^{*a} + \tfrac{1}{2}N_{abA}\eta_{\mu\nu\rho\lambda}^{*a}\Big)\eta^b C^A\Big] \\ & +Q_{aA}(f)\eta^a C^A + \tfrac{1}{3!}Q_{abc}(f)\eta^a\eta^b\eta^c \\ & +\tfrac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}\left({Q^a}_b(f)\eta^b\eta_a^{\alpha\beta\gamma\delta} + Q_a(f)C^{a\alpha\beta\gamma\delta}\right). \end{aligned} \tag{4.49}$$

(Solution (4.49) embeds also the general solution to the homogeneous equation $\gamma\bar{a}_3^{(\mathrm{int})} = 0$.) The elements denoted by $Q_{aA}(f)$, $Q_{abc}(f)$, ${Q^a}_b(f)$, and $Q_a(f)$ are generated via formula (4.25) by the smooth functions (of the undifferentiated scalar fields) f_{aB}^A, f_{abc}^A, ${f^{Aa}}_b$, and f_a^A, respectively. In addition, the functions f_{abc}^A are completely antisymmetric in their BF collection indices.

The interacting component of antighost number 2 results as solution to equation (4.45) for $k = 3$ by relying on formula (4.49) and definitions (2.15)–(2.25), and takes the form

$$\begin{aligned} a_2'^{(\mathrm{int})} = & -\tfrac{1}{2}\left(P_{AB}(N)\right)^{\mu\nu}\left(C^A V_{\mu\nu}^B - \tfrac{1}{2}\varepsilon_{\mu\nu\rho\lambda}C^{A\rho}C^{B\lambda}\right) \\ & -\tfrac{1}{4}\left(P_{abA}(N)\right)^{\mu\nu}\left[\eta^a\eta^b V_{\mu\nu}^A + \varepsilon_{\mu\nu\rho\lambda}\left(2A^{a\rho}\eta^b C^{A\lambda} + A^{a\rho}A^{b\lambda}C^A\right)\right] \\ & +\left(P_A^a(N)\right)_{\mu\nu}\left(C^A B_a^{\mu\nu} + 3C_\rho^A\eta_a^{\mu\nu\rho} + \tfrac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}V^{A\mu\nu}\eta_a^{\alpha\beta\gamma\delta}\right) \\ & -\varepsilon^{\mu\nu\rho\lambda}\Big(\left(P_{abA}(N)\right)_\mu B_{\nu\rho}^{*a} + \tfrac{1}{3}N_{abA}\eta_{\mu\nu\rho}^{*a}\Big)\left(A_\lambda^b C^A + \eta^b C_\lambda^A\right) \end{aligned}$$

$$
\begin{aligned}
&+\tfrac{1}{4!}\varepsilon^{\mu\nu\rho\lambda}\left(Q_a\left(f\right)\right)_\mu C^a_{\nu\rho\lambda}-\left(Q_{aA}\left(f\right)\right)_\mu\left(A^{a\mu}C^A+\eta^a C^{A\mu}\right)\\
&-\tfrac{1}{4!}\left(Q^a{}_b\left(f\right)\right)^\mu\left(\varepsilon_{\alpha\beta\gamma\delta}A^b_\mu\eta_a^{\alpha\beta\gamma\delta}-\varepsilon_{\mu\alpha\beta\gamma}\eta^b\eta_a^{\alpha\beta\gamma}\right)\\
&-\tfrac{1}{2}\left(Q_{abc}\left(f\right)\right)^\mu A^a_\mu\eta^b\eta^c.
\end{aligned}
\tag{4.50}
$$

Using definitions (2.15)–(2.25), we obtain

$$
\delta a_2^{\prime(\mathrm{int})}=\delta c_2+\gamma e_1+\partial_\mu j_1^\mu+h_1,
\tag{4.51}
$$

where

$$
\begin{aligned}
c_2 = {} & \Big(\left(P_{AB}\left(N\right)\right)^\mu C^A+\tfrac{1}{2}\left(P_{abB}\left(N\right)\right)^\mu\eta^a\eta^b\\
&-\varepsilon_{\alpha\beta\gamma\delta}\left(P^a_B\left(N\right)\right)^\mu\eta_a^{\alpha\beta\gamma\delta}\Big)V^{*B}_\mu+2\left(N^a_A\eta^*_a-\left(P^a_A\left(N\right)\right)_\mu A^{*\mu}_a\right)C^A\\
&+\left(\left(Q_{aA}\left(f\right)\right)^{\mu\nu}C^A+\tfrac{1}{2}\left(Q_{abc}\left(f\right)\right)^{\mu\nu}\eta^b\eta^c\right)B^{*a}_{\mu\nu}\\
&+\tfrac{1}{3}\varepsilon^{\mu\nu\rho\lambda}\eta^{*a}_{\mu\nu\rho}V_{B\lambda}\left(f^B_{aA}C^A+\tfrac{1}{2}f^B_{abc}\eta^b\eta^c\right)\\
&-\tfrac{1}{2}\varepsilon^{\mu\nu\rho\lambda}N_{abA}B^{*a}_{\mu\nu}B^{*b}_{\rho\lambda}C^A+\tfrac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}\left(Q^a{}_b\left(f\right)\right)^{\mu\nu}B^{*b}_{\mu\nu}\eta_a^{\alpha\beta\gamma\delta}\\
&-\tfrac{1}{3}f^{Ba}{}_b\eta^{*b}_{\mu\nu\rho}V_{B\lambda}\eta_a^{\mu\nu\rho\lambda},
\end{aligned}
\tag{4.52}
$$

$$
\begin{aligned}
e_1 = {} & A^a_\mu\eta^b\left(\left(P_{abB}\left(N\right)\right)_\nu V^{B\mu\nu}+N_{abB}V^{*B\mu}\right)+2\left(P^a_A\left(N\right)\right)_\mu C^A_\nu B^{\mu\nu}_a\\
&-\varepsilon_{\mu\alpha\beta\gamma}\eta_a^{\alpha\beta\gamma}\left(\left(P^a_A\left(N\right)\right)_\nu V^{A\mu\nu}+N^a_B V^{*B\mu}\right)-2N^a_A A^{*\mu}_a C^A_\mu\\
&+N_{abA}B^{*a}_{\mu\nu}\eta^b V^{A\mu\nu}-\varepsilon^{\mu\nu\rho\lambda}\left(\tfrac{1}{2}\left(P_{abA}\left(N\right)\right)_\mu A^a_\nu+N_{abA}B^{*a}_{\mu\nu}\right)A^b_\rho C^A_\lambda\\
&-C^A_\mu\left(\left(P_{AB}\left(N\right)\right)_\nu V^{B\mu\nu}+N_{AB}V^{*B\mu}\right)-\varepsilon^{\mu\nu\rho\lambda}f^B_{aA}B^{*a}_{\mu\nu}V_{B\rho}C^A_\lambda\\
&+\left(Q_{aA}\left(f\right)\right)^{\mu\nu}\left(A^a_\mu C^A_\nu+\tfrac{1}{4}\varepsilon_{\mu\nu\rho\lambda}\eta^a V^{A\rho\lambda}\right)-\tfrac{1}{2}\left(Q_{abc}\left(f\right)\right)^{\mu\nu}A^a_\mu A^b_\nu\eta^c\\
&+\varepsilon^{\mu\nu\rho\lambda}f^B_{abc}B^{*a}_{\mu\nu}V_{B\rho}A^b_\lambda\eta^c+\tfrac{1}{2\cdot 4!}\varepsilon^{\mu\nu\rho\lambda}\left(Q_a\left(f\right)\right)_{\mu\nu}C^a_{\rho\lambda}\\
&+\tfrac{1}{4!}\left(Q^a{}_b\left(f\right)\right)^{\mu\nu}\left(\tfrac{1}{2}\varepsilon_{\mu\nu\rho\lambda}\eta^b B_a^{\rho\lambda}-\varepsilon_{\nu\alpha\beta\gamma}A^b_\mu\eta_a^{\alpha\beta\gamma}\right)\\
&+\tfrac{1}{4}f^{Ba}{}_b B^{*b}_{\mu\nu}V_{B\rho}\eta_a^{\mu\nu\rho},
\end{aligned}
\tag{4.53}
$$

$$
\begin{aligned}
j_1^\mu = {} & -\left(N_{AB}C^A+\tfrac{1}{2}N_{abB}\eta^a\eta^b-\varepsilon_{\alpha\beta\gamma\delta}N^a_B\eta_a^{\alpha\beta\gamma\delta}\right)V^{*B\mu}+2\left(N^a_A A^{*\mu}_a\right.\\
&\left.+\left(P^a_A\left(N\right)\right)_\nu B^{\mu\nu}_a\right)C^A+\left(P^a_A\left(N\right)\right)_\nu\left(6C^A_\rho\eta_a^{\mu\nu\rho}+\varepsilon_{\alpha\beta\gamma\delta}V^{A\mu\nu}\eta_a^{\alpha\beta\gamma\delta}\right)\\
&-\left(P_{AB}\left(N\right)\right)_\nu\left(C^A V^{B\mu\nu}-\tfrac{1}{2}\varepsilon^{\mu\nu\rho\lambda}C^A_\rho C^B_\lambda\right)\\
&-\varepsilon^{\mu\nu\rho\lambda}N_{abA}B^{*a}_{\nu\rho}\left(\eta^b C^A_\lambda+A^b_\lambda C^A\right)-\tfrac{1}{2}\left(P_{abA}\left(N\right)\right)_\nu\eta^a\eta^b V^{A\mu\nu}\\
&-\varepsilon^{\mu\nu\rho\lambda}\left(P_{abA}\left(N\right)\right)_\nu A^a_\rho\left(\eta^b C^A_\lambda+\tfrac{1}{2}A^b_\lambda C^A\right)+f^{Ba}{}_b B^{*b}_{\nu\rho}V_{B\lambda}\eta_a^{\mu\nu\rho\lambda}
\end{aligned}
$$

$$
\begin{aligned}
&+ (Q_{aA}(f))^{\mu\nu} \left(A^a_\nu C^A + \eta^a C^A_\nu\right) + \tfrac{1}{2} (Q_{abc}(f))^{\mu\nu} A^a_\nu \eta^b \eta^c \\
&-\varepsilon^{\mu\nu\rho\lambda} B^{*a}_{\nu\rho} V_{B\lambda} \left(f^B_{aA} C^A + \tfrac{1}{2} f^B_{abc} \eta^b \eta^c\right) - \tfrac{1}{4!} \varepsilon_{\nu\alpha\beta\gamma} (Q_a(f))^{\mu\nu} C^{a\alpha\beta\gamma} \\
&+\tfrac{1}{4!} (Q^a{}_b(f))^{\mu\nu} \left(\varepsilon_{\alpha\beta\gamma\delta} A^b_\nu \eta^{\alpha\beta\gamma\delta}_a - \varepsilon_{\nu\alpha\beta\gamma} \eta^b \eta^{\alpha\beta\gamma}_a\right),
\end{aligned}
\tag{4.54}
$$

$$
\begin{aligned}
h_1 \;=\;& \left((P_{AB}(N))^\mu C^A + \tfrac{1}{2} (P_{abB}(N))^\mu \eta^a \eta^b - \varepsilon_{\alpha\beta\gamma\delta} (P^a_B(N))^\mu \eta^{\alpha\beta\gamma\delta}_a\right) V^B_\mu \\
&+ \left(N_{AB} C^A + \tfrac{1}{2} N_{abB} \eta^a \eta^b - \varepsilon_{\alpha\beta\gamma\delta} N^a_B \eta^{\alpha\beta\gamma\delta}_a\right) \partial^\mu V^{*B}_\mu .
\end{aligned}
\tag{4.55}
$$

If we make the notation

$$
a_2^{(\mathrm{int})} \equiv a_2'^{(\mathrm{int})} - c_2, \tag{4.56}
$$

then (4.51) is equivalent with the equation

$$
\delta a_2^{(\mathrm{int})} = \gamma e_1 + \partial_\mu j_1^\mu + h_1. \tag{4.57}
$$

Comparing (4.57) with equation (4.45) for $k = 2$, we obtain that a necessary condition for the existence of a local $a_1^{(\mathrm{int})}$ is

$$
h_1 = \delta g_2 + \gamma f_1 + \partial_\mu l_1^\mu, \tag{4.58}
$$

with g_2, f_1, and l_1^μ local functions. We show that equation (4.58) cannot hold (locally) unless $h_1 = 0$. Indeed, assuming (4.58) is satisfied, we act with δ on it and use its nilpotency and anticommutation with γ, which yields the necessary condition

$$
\delta h_1 = \gamma(-\delta f_1) + \partial_\mu \left(\delta l_1^\mu\right). \tag{4.59}
$$

On the other hand, direct computation provides

$$
\begin{aligned}
\delta h_1 \;=\;& \gamma \left[\left(N_{AB} C^A_\mu - N_{abB} A^a_\mu \eta^b + \varepsilon_{\mu\alpha\beta\gamma} N^a_B \eta^{\alpha\beta\gamma}_a\right) V^{B\mu}\right] \\
&+\partial_\mu \left[-\left(N_{AB} C^A + \tfrac{1}{2} N_{abB} \eta^a \eta^b - \varepsilon_{\alpha\beta\gamma\delta} N^a_B \eta^{\alpha\beta\gamma\delta}_a\right) V^{B\mu}\right].
\end{aligned}
\tag{4.60}
$$

Juxtaposing (4.59) and (4.60) and looking at definitions (2.15)–(2.25), it follows that $V^{B\mu}$ must necessarily be δ-exact modulo d in the space of local functions. Since this is obviously not true, we find that (4.59) cannot be satisfied and consequently neither does equation (4.58). Thus, the consistency of $a_2^{(\mathrm{int})}$ leads to the equation

$$
h_1 = 0, \tag{4.61}
$$

which further implies that the functions N_{abA}, N_{AB}, and N^a_A must vanish

$$
N_{abA} = N_{AB} = N^a_A = 0. \tag{4.62}
$$

Based on (4.62), from (4.47), (4.49), (4.50), (4.52), (4.53), (4.56), and (4.57) we get the components of antighost number 4, 3, and 2 from the nonintegrated density of the first-order deformation as

$$a_4^{(\mathrm{int})} = 0, \tag{4.63}$$

$$\begin{aligned} a_3^{(\mathrm{int})} &= Q_{aA}(f)\,\eta^a C^A + \tfrac{1}{3!} Q_{abc}(f)\,\eta^a\eta^b\eta^c \\ &\quad + \tfrac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}\left(Q^a{}_b(f)\,\eta^b\eta_a^{\alpha\beta\gamma\delta} + Q_a(f)\,C^{a\alpha\beta\gamma\delta}\right), \end{aligned} \tag{4.64}$$

$$\begin{aligned} a_2^{(\mathrm{int})} &= \tfrac{1}{4!}\varepsilon^{\mu\nu\rho\lambda}\left(Q_a(f)\right)_\mu C^a_{\nu\rho\lambda} - \left(Q_{aA}(f)\right)^\mu \left(A^a_\mu C^A + \eta^a C^A_\mu\right) \\ &\quad - \tfrac{1}{2}\left(Q_{abc}(f)\right)^\mu A^a_\mu \eta^b\eta^c - \tfrac{1}{4!}\left(Q^a{}_b(f)\right)^\mu \left(\varepsilon_{\alpha\beta\gamma\delta}A^b_\mu \eta_a^{\alpha\beta\gamma\delta}\right. \\ &\quad \left. - \varepsilon_{\mu\alpha\beta\gamma}\eta^b\eta_a^{\alpha\beta\gamma}\right) - \left(\left(Q_{aA}(f)\right)^{\mu\nu} C^A + \tfrac{1}{2}\left(Q_{abc}(f)\right)^{\mu\nu}\eta^b\eta^c\right) B^{*a}_{\mu\nu} \\ &\quad - \tfrac{1}{3}\varepsilon^{\mu\nu\rho\lambda}\eta^{*a}_{\mu\nu\rho}V_{B\lambda}\left(f^B_{aA}C^A + \tfrac{1}{2}f^B_{abc}\eta^b\eta^c\right) + \tfrac{1}{3}f^{Ba}{}_b\,\eta^{*b}_{\mu\nu\rho}V_{B\lambda}\eta_a^{\mu\nu\rho\lambda} \\ &\quad - \tfrac{1}{4!}\varepsilon_{\alpha\beta\gamma\delta}\left(Q^a{}_b(f)\right)^{\mu\nu} B^{*b}_{\mu\nu}\eta_a^{\alpha\beta\gamma\delta} + \tfrac{1}{2}R_{ab}(g)\,\eta^a\eta^b \\ &\quad + R_A(g)\,C^A + \tfrac{1}{4!}\varepsilon_{\mu\nu\rho\lambda}R^a(g)\,\eta_a^{\mu\nu\rho\lambda}. \end{aligned} \tag{4.65}$$

The objects $R_{ab}(g)$, $R_A(g)$, and $R^a(g)$ are generated by formula (4.30) via the smooth functions of the undifferentiated scalar fields g^{AB}_{ab}, $g^{AB}{}_C$, and g^{aAB}, respectively. All these functions are antisymmetric in A and B and in addition g^{AB}_{ab} are antisymmetric also in their (lower) BF collection indices.

Replacing now expression (4.65) into equation (4.45) for $k = 2$, we obtain that the interacting piece of antighost number 1 from the first-order deformation is written as

$$\begin{aligned} a_1'^{(\mathrm{int})} &= -\tfrac{1}{2\cdot 4!}\varepsilon^{\mu\nu\rho\lambda}\left(Q_a(f)\right)_{\mu\nu} C^a_{\rho\lambda} - \left(Q_{aA}(f)\right)^{\mu\nu}\left(A^a_\mu C^A_\nu\right. \\ &\quad \left. + \tfrac{1}{4}\varepsilon_{\mu\nu\rho\lambda}\eta^a V^{A\rho\lambda}\right) + \tfrac{1}{4!}\left(Q^a{}_b(f)\right)^{\mu\nu}\left(\varepsilon_{\nu\alpha\beta\gamma}A^b_\mu\eta_a^{\alpha\beta\gamma} - \tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\eta^b B_a^{\alpha\beta}\right) \\ &\quad + \left(R_A(g)\right)^\mu C^A_\mu - \left(R_{ab}(g)\right)^\mu A^a_\mu\eta^b - \tfrac{1}{4!}\varepsilon_{\mu\nu\rho\lambda}\left(R^a(g)\right)^\mu\eta_a^{\nu\rho\lambda} \\ &\quad + \varepsilon^{\mu\nu\rho\lambda}B^{*a}_{\mu\nu}V_{B\rho}\left(f^B_{aA}C^A_\lambda - f^B_{abc}A^b_\lambda\eta^c - \tfrac{1}{4!}\varepsilon_{\lambda\alpha\beta\gamma}f^{Bb}{}_a\,\eta_b^{\alpha\beta\gamma}\right) \\ &\quad + \tfrac{1}{2}\left(Q_{abc}(f)\right)^{\mu\nu}A^a_\mu A^b_\nu\eta^c. \end{aligned} \tag{4.66}$$

Using definitions (2.15)–(2.25), by direct computation we obtain that

$$\delta a_1'^{(\mathrm{int})} = \delta c_1 + \gamma e_0 + \partial_\mu j_0^\mu + h_0, \tag{4.67}$$

with

$$c_1 = -\eta^a V_{B\mu} \left(f^B_{aA} V^{*A\mu} + \tfrac{1}{12} f^{Bb}_{\ \ \ a} A^{*\mu}_b + \tfrac{1}{2} \varepsilon^{\mu\nu\rho\lambda} g^{AB}_{ab} V_{A\nu} B^{*b}_{\rho\lambda} \right), \tag{4.68}$$

$$\begin{aligned} e_0 &= -\tfrac{1}{2} \varepsilon^{\mu\nu\rho\lambda} V_{A\mu} \left(-\tfrac{1}{3} f^A_{abc} A^c_\nu + \tfrac{1}{2} g^{AB}_{ab} V_{B\nu} \right) A^a_\rho A^b_\lambda \\ &\quad + \tfrac{1}{4!} f^A_a V^\mu_A H^a_\mu - A^a_\mu V_{A\nu} \left(f^A_{aB} V^{B\mu\nu} + \tfrac{1}{12} f^{Ab}_{\ \ \ a} B^{\mu\nu}_b \right) \\ &\quad -\tfrac{1}{2} \left(g^{AB}_{\ \ \ \ C} V^C_{\mu\nu} + \tfrac{1}{12} g^{aAB} B_{a\mu\nu} \right) V^\mu_A V^\nu_B, \end{aligned} \tag{4.69}$$

$$\begin{aligned} j^\mu_0 &= V_{A\nu} \left(\tfrac{1}{12} f^A_a C^{a\mu\nu} + f^A_{aB} \eta^a V^{B\mu\nu} \right) + \tfrac{1}{4} f^{Aa}_{\ \ \ \ b} V_{A\nu} \left(A^b_\rho \eta^{\mu\nu\rho}_a \right. \\ &\quad \left. + \tfrac{1}{3} \eta^b B^{\mu\nu}_a \right) - \tfrac{1}{8} g^{aAB} V_{A\nu} V_{B\rho} \eta^{\mu\nu\rho}_a - \varepsilon^{\mu\nu\rho\lambda} \left[f^A_{aB} A^a_\nu V_{A\lambda} C^B_\rho \right. \\ &\quad \left. -\tfrac{1}{2} f^A_{abc} A^a_\nu A^b_\rho \eta^c V_{A\lambda} - \tfrac{1}{2} V_{A\nu} V_{B\rho} \left(g^{AB}_{\ \ \ \ C} C^C_\lambda - g^{AB}_{ab} A^a_\lambda \eta^b \right) \right], \end{aligned} \tag{4.70}$$

$$h_0 = -f^A_{aB} \eta^a V^\mu_A V^B_\mu. \tag{4.71}$$

At this stage we act like between formulas (4.56) and (4.62). If we make the notation

$$a^{(\mathrm{int})}_1 = a'^{(\mathrm{int})}_1 - c_1, \tag{4.72}$$

then (4.67) becomes

$$\delta a^{(\mathrm{int})}_1 = \gamma e_0 + \partial_\mu j^\mu_0 + h_0, \tag{4.73}$$

which, compared with equation (4.45) for $k = 1$, reveals that the existence of $a^{(\mathrm{int})}_0$ demands

$$h_0 = \delta g_1 + \gamma f_0 + \partial_\mu l^\mu_0, \tag{4.74}$$

with g_1, f_0, and l^μ_0 some local elements. Using (4.71) and definitions (2.15)–(2.25), straightforward calculation shows that (4.74) cannot be valid, and hence the consistency of $a^{(\mathrm{int})}_1$ leads to the equation

$$h_0 = 0, \tag{4.75}$$

which requires the antisymmetry of the functions f_{aAB} ($\equiv k_{AM} f^M_{aB}$) with respect to their collection indices from the two-form sector

$$f_{aAB} = -f_{aBA}. \tag{4.76}$$

With the help of (4.66), (4.68), (4.69), (4.72), (4.73), and (4.76) we completely determine $a^{(\mathrm{int})}_1$ and then $a^{(\mathrm{int})}_0$ as solution to (4.45) for $k = 1$ in the form

$$a^{(\mathrm{int})}_1 = -\tfrac{1}{2\cdot 4!} \varepsilon^{\mu\nu\rho\lambda} \left(Q_a(f) \right)_{\mu\nu} C^a_{\rho\lambda} - \left(Q_{aA}(f) \right)^{\mu\nu} \left(A^a_\mu C^A_\nu \right.$$

$$
\begin{aligned}
&+\tfrac{1}{4}\varepsilon_{\mu\nu\rho\lambda}\eta^{a}V^{A\rho\lambda}\Big)+\tfrac{1}{4!}\left(Q^{a}{}_{b}\left(f\right)\right)^{\mu\nu}\left(\varepsilon_{\nu\alpha\beta\gamma}A_{\mu}^{b}\eta_{a}^{\alpha\beta\gamma}-\tfrac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\eta^{b}B_{a}^{\alpha\beta}\right)\\
&+\left(R_{A}\left(g\right)\right)^{\mu}C_{\mu}^{A}-\left(R_{ab}\left(g\right)\right)^{\mu}A_{\mu}^{a}\eta^{b}-\tfrac{1}{4!}\varepsilon_{\mu\nu\rho\lambda}\left(R^{a}\left(g\right)\right)^{\mu}\eta_{a}^{\nu\rho\lambda}\\
&+\varepsilon^{\mu\nu\rho\lambda}B_{\mu\nu}^{*a}V_{B\rho}\left(f_{aA}^{B}C_{\lambda}^{A}-f_{abc}^{B}A_{\lambda}^{b}\eta^{c}-\tfrac{1}{4!}\varepsilon_{\lambda\alpha\beta\gamma}f^{Bb}{}_{a}\eta_{b}^{\alpha\beta\gamma}\right)\\
&+\tfrac{1}{2}\left(Q_{abc}\left(f\right)\right)^{\mu\nu}A_{\mu}^{a}A_{\nu}^{b}\eta^{c}+\eta^{a}V_{B\mu}\Big(f_{aA}^{B}V^{*A\mu}+\tfrac{1}{12}f^{Bb}{}_{a}A_{b}^{*\mu}\\
&+\tfrac{1}{2}\varepsilon^{\mu\nu\rho\lambda}g_{ab}^{AB}V_{A\nu}B_{\rho\lambda}^{*b}\Big), \qquad (4.77)
\end{aligned}
$$

$$
\begin{aligned}
a_{0}^{(\mathrm{int})} &= \tfrac{1}{2}\varepsilon^{\mu\nu\rho\lambda}V_{A\mu}\left(-\tfrac{1}{3}f_{abc}^{A}A_{\nu}^{c}+\tfrac{1}{2}g_{ab}^{AB}V_{B\nu}\right)A_{\rho}^{a}A_{\lambda}^{b}\\
&-\tfrac{1}{4!}f_{a}^{A}V_{A}^{\mu}H_{\mu}^{a}+f_{aB}^{A}A_{\mu}^{a}V_{A\nu}V^{B\mu\nu}+\tfrac{1}{12}f^{Ab}{}_{a}A_{\mu}^{a}V_{A\nu}B_{b}^{\mu\nu}\\
&+\tfrac{1}{2}\left(g^{AB}{}_{C}V_{\mu\nu}^{C}+\tfrac{1}{12}g^{aAB}B_{a\mu\nu}\right)V_{A}^{\mu}V_{B}^{\nu}. \qquad (4.78)
\end{aligned}
$$

Thus, we can write the final form of the interacting part from the first-order deformation of the solution to the master equation for a collection of BF models and a set of two-form gauge fields as

$$
S_{1}^{(\mathrm{int})}\equiv\int d^{4}x\,a^{(\mathrm{int})}=\int d^{4}x\left(a_{3}^{(\mathrm{int})}+a_{2}^{(\mathrm{int})}+a_{1}^{(\mathrm{int})}+a_{0}^{(\mathrm{int})}\right), \qquad (4.79)
$$

where the 4 components from (4.79) read as in formulas (4.64)–(4.65) and (4.77)–(4.78), respectively. The previous first-order deformation is parameterized by 7 functions, f_{abc}^{A}, g_{ab}^{AB}, f_{a}^{A}, f_{aB}^{A}, $f^{Ab}{}_{a}$, $g^{AB}{}_{C}$, and g^{aAB}, which depend smoothly on the undifferentiated scalar fields φ_{d} and are antisymmetric as follows: f_{abc}^{A} in the indices $\{a,b,c\}$, g_{ab}^{AB} with respect to $\{A,B\}$ and $\{a,b\}$, and $f_{aAB}\equiv k_{AM}f_{aB}^{M}$ together with $g^{AB}{}_{C}$ and g^{aAB} in $\{A,B\}$. It is easy to see that (4.79) also includes the general solution that describes the self-interactions among the two-form gauge fields. Indeed, if we isolate from $S_{1}^{(\mathrm{int})}$ the part containing the functions $g^{AB}{}_{C}$, represent these functions as some series in the undifferentiated scalar fields, $g^{AB}{}_{C}\left(\varphi_{a}\right)=k^{AB}{}_{C}+k^{ABa}_{C}\varphi_{a}+\cdots$, where $k^{AB}{}_{C}$ and k^{ABa}_{C} are some real constants, antisymmetric in their upper, capital indices, and retain only the terms including $k^{AB}{}_{C}$, then we obtain

$$
\begin{aligned}
S_{1}^{(\mathrm{int})}(k) &\equiv \int d^{4}x\,a^{(\mathrm{V})}=\int d^{4}x\left(a_{2}^{(\mathrm{V})}+a_{1}^{(\mathrm{V})}+a_{0}^{(\mathrm{V})}\right)\\
&= k^{AB}{}_{C}\int d^{4}x\Big[\left(C_{A}^{*\mu}V_{B\mu}+\tfrac{1}{2}\varepsilon_{\mu\nu\rho\lambda}V_{A}^{*\mu\nu}V_{B}^{*\rho\lambda}\right)C^{C}\\
&\quad+\varepsilon_{\mu\nu\rho\lambda}V_{A}^{*\mu\nu}V_{B}^{\rho}C^{C\lambda}+\tfrac{1}{2}V_{\mu\nu}^{C}V_{A}^{\mu}V_{B}^{\nu}\Big], \qquad (4.80)
\end{aligned}
$$

which has been shown in [62] to be the most general form of the first-order deformation for a set of two-form gauge fields in four spacetime dimensions with the Lagrangian action written in first-order form. In conclusion, the overall first-order deformation of the solution to the master equation for the model under study is expressed like the sum between (4.79) and the piece responsible for the interactions from the BF sector

$$S_1 = S_1^{(\mathrm{BF})} + S_1^{(\mathrm{int})}, \tag{4.81}$$

where

$$S_1^{(\mathrm{BF})} = \int d^4x \, a^{(\mathrm{BF})}, \tag{4.82}$$

with $a^{(\mathrm{BF})}$ provided by (4.36) and (4.37)–(4.41). We recall that $S_1^{(\mathrm{BF})}$ is parameterized by 4 kinds of smooth functions of the undifferentiated scalar fields: W_{ab}, M_{ab}^{c}, M^{ab}, and M_{abcd}, where M_{ab}^{c} are antisymmetric in their lower indices, M^{ab} are symmetric, and M_{abcd} are completely antisymmetric.

4.3 Second-order deformation

Next, we investigate the equations responsible for higher-order deformations. The second-order deformation is governed by equation (3.5). Making use of the first-order deformation derived in the previous subsection, after some computation we organize the second term on the left-hand side of (3.5) like

$$(S_1, S_1) = \int d^4x \left(\Delta + \bar{\Delta}\right), \tag{4.83}$$

where

$$\begin{aligned} \Delta \;=\; & \sum_{p=0}^{4} \left(K^{abc}_{,m_1 \ldots m_p} \frac{\partial^p t_{abc}}{\partial \varphi_{m_1} \ldots \partial \varphi_{m_p}} + K^{abc}_{d,m_1 \ldots m_p} \frac{\partial^p t^{d}_{abc}}{\partial \varphi_{m_1} \ldots \partial \varphi_{m_p}} \right. \\ & + K^{abcdf}_{m_1 \ldots m_p} \frac{\partial^p t_{abcdf}}{\partial \varphi_{m_1} \ldots \partial \varphi_{m_p}} + K^{a}_{b,m_1 \ldots m_p} \frac{\partial^p t^{b}_{a}}{\partial \varphi_{m_1} \ldots \partial \varphi_{m_p}} \\ & \left. + K^{c}_{ab,m_1 \ldots m_p} \frac{\partial^p t^{ab}_{c}}{\partial \varphi_{m_1} \ldots \partial \varphi_{m_p}} \right) \end{aligned} \tag{4.84}$$

and

$$\bar{\Delta} \;=\; \sum_{p=0}^{3} \left(X^{abB}_{A,m_1 \ldots m_p} \frac{\partial^p T^{A}_{abB}}{\partial \varphi_{m_1} \ldots \partial \varphi_{m_p}} + X^{abcd}_{A,m_1 \ldots m_p} \frac{\partial^p T^{A}_{abcd}}{\partial \varphi_{m_1} \ldots \partial \varphi_{m_p}} \right.$$

$$
\begin{aligned}
&+X^{ab}_{A,m_1\ldots m_p}\frac{\partial^p T^A_{ab}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}+X^{ab}_{Ac,m_1\ldots m_p}\frac{\partial^p T^{Ac}_{ab}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}\\
&+X_{Aab,m_1\ldots m_p}\frac{\partial^p T^{Aab}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}+X^{AB}_{a,m_1\ldots m_p}\frac{\partial^p T^a_{AB}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}\Bigg)\\
&+\sum_{p=0}^{2}\Bigg(X^{aABC}_{m_1\ldots m_p}\frac{\partial^p T_{aABC}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}+X^{abc}_{AB,m_1\ldots m_p}\frac{\partial^p T^{AB}_{abc}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}\\
&+X^a_{AB,m_1\ldots m_p}\frac{\partial^p T^{AB}_a}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}+X^b_{ABa,m_1\ldots m_p}\frac{\partial^p T^{ABa}_b}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}\Bigg)\\
&+\sum_{p=0}^{1}\Bigg(X_{ABCD,m_1\ldots m_p}\frac{\partial^p T^{ABCD}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}+X^{ab}_{ABC,m_1\ldots m_p}\frac{\partial^p T^{ABC}_{ab}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}\\
&+X^{ABC}_{a,m_1\ldots m_p}\frac{\partial^p T^a_{ABC}}{\partial\varphi_{m_1}\ldots\partial\varphi_{m_p}}\Bigg)+X^a_{ABCD}T^{ABCD}_a. \qquad (4.85)
\end{aligned}
$$

In formulas (4.84) and (4.85) we used the notations

$$
\begin{aligned}
t_{abc} &= W_{ec}M^e_{ab}+W_{ea}\frac{\partial W_{bc}}{\partial\varphi_e}+W_{eb}\frac{\partial W_{ca}}{\partial\varphi_e}, && (4.86)\\
t^d_{abc} &= W_{e[a}\frac{\partial M^d_{bc]}}{\partial\varphi_e}+M^d_{e[a}M^e_{bc]}+M^{de}M_{eabc}, && (4.87)\\
t_{abcdf} &= W_{e[a}\frac{\partial M_{bcdf]}}{\partial\varphi_e}+M_{e[abc}M^e_{df]}, && (4.88)\\
t^b_a &= M^{be}W_{ea}, && (4.89)\\
t^{bc}_a &= W_{ea}\frac{\partial M^{bc}}{\partial\varphi_e}+M^{(b}_{ea}M^{c)e}, && (4.90)
\end{aligned}
$$

$$
T^A_{ab}=f^A_{aM}f^M_b+f^A_e\frac{\partial W_{ab}}{\partial\varphi_e}+W_{ea}\frac{\partial f^A_b}{\partial\varphi_e}+2W_{eb}f^{\ Ae}_{\ a}, \qquad (4.91)
$$

$$
T^{AB}_a=f^A_e\frac{\partial f^B_a}{\partial\varphi_e}-f^B_e\frac{\partial f^A_a}{\partial\varphi_e}-4!\left(g^{AB}{}_M f^M_a+2W_{ea}g^{eAB}\right), \qquad (4.92)
$$

$$
\begin{aligned}
T^{Ac}_{ab} &= f^A_{aM}f^{\ Mc}_{\ b}-f^A_{bM}f^{\ Mc}_{\ a}-\tfrac{1}{2}f^A_e\frac{\partial M^c_{ab}}{\partial\varphi_e}+f^{\ Ac}_{\ e}M^e_{ab}\\
&\quad +f^{\ Ae}_{\ [a}M^c_{b]e}-2\cdot 4!f^A_{eab}M^{ec}+W_{e[a}\frac{\partial f^{\ Ac}_{\ b]}}{\partial\varphi_e}, \qquad (4.93)
\end{aligned}
$$

$$
\begin{aligned}
T^{A}_{abcd} &= W_{e[a}\frac{\partial f^{A}_{bcd]}}{\partial\varphi_e} + f^{A}_{e[ab}M^{e}_{cd]} + f^{A}_{M[a}f^{M}_{bcd]} \\
&\quad + \tfrac{1}{2\cdot 4!}\left(\tfrac{1}{2}f^{A}_{e}\frac{\partial M_{abcd}}{\partial\varphi_e} - f^{Ae}{}_{[a}M_{bcd]e}\right),
\end{aligned} \tag{4.94}
$$

$$
T^{Aab} = f^{A}_{e}\frac{\partial M^{ab}}{\partial\varphi_e} - 2f^{Aa}{}_{e}M^{be} - 2f^{Ab}{}_{e}M^{ae}, \tag{4.95}
$$

$$
T^{A}_{abB} = f^{A}_{M[a}f^{M}_{b]B} + f^{A}_{eB}M^{e}_{ab} + W_{e[a}\frac{\partial f^{A}_{b]B}}{\partial\varphi_e}, \tag{4.96}
$$

$$
\begin{aligned}
T_{aABC} &= f_{Ae}\frac{\partial f_{aBC}}{\partial\varphi_e} - f_{Be}\frac{\partial f_{aAC}}{\partial\varphi_e} + 2f^{e}{}_{Aa}f_{eBC} - 2f^{e}{}_{Ba}f_{eAC} \\
&\quad + 4!\left(-g_{ABM}f^{M}_{aC} + W_{ea}\frac{\partial g_{ABC}}{\partial\varphi_e} + f^{M}_{a[A}g_{B]MC}\right),
\end{aligned} \tag{4.97}
$$

$$
T^{a}_{AB} = f_{eAB}M^{ea}, \tag{4.98}
$$

$$
\begin{aligned}
T^{AB}_{abc} &= f^{A}_{e}\frac{\partial f^{B}_{abc}}{\partial\varphi_e} - f^{B}_{e}\frac{\partial f^{A}_{abc}}{\partial\varphi_e} + 2f^{Ae}{}_{[a}f^{B}_{bc]e} - 2f^{Be}{}_{[a}f^{A}_{bc]e} \\
&\quad + \tfrac{1}{2}g^{eAB}M_{abce} + 4!\left(g^{AB}_{e[a}M^{e}_{bc]} + W_{e[a}\frac{\partial g^{AB}_{bc]}}{\partial\varphi_e}\right) \\
&\quad - 4!\left(g^{AB}{}_{M}f^{M}_{abc} + f^{[A}_{M[a}g^{B]M}_{bc]}\right),
\end{aligned} \tag{4.99}
$$

$$
\begin{aligned}
T^{ABa}_{b} &= f^{A}_{e}\frac{\partial f^{Ba}{}_{b}}{\partial\varphi_e} - f^{B}_{e}\frac{\partial f^{Aa}{}_{b}}{\partial\varphi_e} - 2f^{Aa}{}_{e}f^{Be}{}_{b} + 2f^{Ba}{}_{e}f^{Ae}{}_{b} \\
&\quad + 4!\left(g^{eAB}M^{a}_{eb} + W_{eb}\frac{\partial g^{aAB}}{\partial\varphi_e}\right) - 4!\left(g^{AB}{}_{M}f^{Ma}{}_{b}\right. \\
&\quad \left. + 2\cdot 4!g^{AB}_{eb}M^{ea} + f^{A}_{bM}g^{aBM} - f^{B}_{bM}g^{aAM}\right),
\end{aligned} \tag{4.100}
$$

$$
T^{ABCD} = g^{e[AB}f^{C]D}_{e} - \tfrac{1}{2}f^{[A}_{e}\frac{\partial g^{BC]D}}{\partial\varphi_e} - 12g^{[AB}{}_{M}g^{C]MD}, \tag{4.101}
$$

$$
T^{ABC}_{ab} = g^{e[AB}f^{C]}_{eab} - \tfrac{1}{2}f^{[A}_{e}\frac{\partial g^{BC]}_{ab}}{\partial\varphi_e} - 12g^{[AB}{}_{M}g^{C]M}_{ab} + g^{[AB}_{e[a}f^{C]e}{}_{b]}, \tag{4.102}
$$

$$T^{a}_{ABC} = g^{e}_{[AB}f^{a}{}_{C]e} - \tfrac{1}{2}f_{e[A}\frac{\partial g^{a}_{BC]}}{\partial\varphi_{e}} - 12g_{[AB}{}^{M}g^{a}_{C]M}, \tag{4.103}$$

$$T^{ABCD}_{a} = g^{e[AB}g^{CD]}_{ea}, \tag{4.104}$$

where the functions g_{ABC}, g^{CMD}, and $g_{AB}{}^{M}$ result from $g^{AB}{}_{M}$ by appropriately lowering or raising the two-form collection indices with the help of the metric k_{AB} or its inverse k^{AB}: $g_{ABC} = k_{AM}k_{BN}g^{MN}{}_{C}$, $g^{CMD} = g^{CM}{}_{E}k^{ED}$, $g_{AB}{}^{M} = k_{AE}k_{BF}g^{EF}{}_{N}k^{NM}$. The remaining objects, of the type K or X, are listed in Appendix A. Each of them is a polynomial of ghost number 1 involving only the *undifferentiated* fields/ghosts and antifields. Comparing equation (3.5) with (4.83), we obtain that the existence of S_2 requires that $\int d^4x\left(\Delta + \bar{\Delta}\right)$ is s-exact. This is not possible since all the objects denoted by K or X are polynomials comprising only undifferentiated fields/ghosts/antifields, so (3.5) takes place if and only if the following equations are simultaneously obeyed

$$t_{abc} = 0, \quad t^{d}_{abc} = 0, \quad t_{abcdf} = 0, \quad t^{b}_{a} = 0, \quad t^{bc}_{a} = 0, \tag{4.105}$$

$$T^{A}_{ab} = 0, \quad T^{AB}_{a} = 0, \quad T^{Ac}_{ab} = 0, \quad T^{A}_{abcd} = 0, \quad T^{Aab} = 0, \tag{4.106}$$

$$T^{A}_{abB} = 0, \quad T_{aABC} = 0, \quad T^{a}_{AB} = 0, \quad T^{AB}_{abc} = 0, \quad T^{ABa}_{b} = 0, \tag{4.107}$$

$$T^{ABCD} = 0, \quad T^{ABC}_{ab} = 0, \quad T^{a}_{ABC} = 0, \quad T^{ABCD}_{a} = 0. \tag{4.108}$$

Based on the last equations, which enforce $\Delta = 0 = \bar{\Delta}$, from (4.83) compared with (3.5) it follows that we can take

$$S_2 = 0. \tag{4.109}$$

On behalf of (4.109) it is easy to show that one can safely set zero the solutions to the higher-order deformation equations, (3.6), etc.

$$S_k = 0, \quad k > 2. \tag{4.110}$$

Collecting formulas (4.109) and (4.110), we can state that the complete deformed solution to the master equation for the model under study, which is consistent to all orders in the coupling constant, reads as

$$S = \bar{S} + \lambda S_1, \tag{4.111}$$

where $\bar{S}$ is given in (2.26) and S_1 is expressed by (4.81). The full deformed solution to the master equation comprises 11 types of smooth functions of the undifferentiated scalar fields: W_{ab}, M^{a}_{bc}, M_{abcd}, M^{ab}, f^{A}_{abc}, g^{AB}_{ab}, f^{A}_{a}, f^{A}_{aB}, f^{Ab}_{a}, $g^{AB}{}_{C}$, and g^{aAB}. They are subject to equations (4.105)–(4.108), imposed by the consistency of the first-order deformation.

5 Lagrangian formulation of the interacting model

The piece of antighost number 0 from the full deformed solution to the master equation, of the form (4.111), furnishes us with the Lagrangian action of the interacting theory

$$
\begin{aligned}
S^{\mathrm{L}}[A_{\mu}^{a}, H_{\mu}^{a}, \varphi_{a}, B_{a}^{\mu\nu}, V_{\mu\nu}^{A}, V_{\mu}^{A}] = & \int d^{4}x \left[H_{\mu}^{a} D^{\mu}\varphi_{a} + \tfrac{1}{2} B_{a}^{\mu\nu} \bar{F}_{\mu\nu}^{a}\right. \\
& + \tfrac{1}{2}\left(V_{A}^{\mu\nu} \bar{F}_{\mu\nu}^{A} + V_{\mu}^{A} V_{A}^{\mu}\right) \\
& - \tfrac{\lambda}{4} \varepsilon^{\mu\nu\rho\lambda}\left(\tfrac{1}{4!} M_{abcd} A_{\mu}^{a} A_{\nu}^{b} + \tfrac{2}{3} f_{Aacd} V_{\mu}^{A} A_{\nu}^{a}\right. \\
& \left.\left. - g_{ABcd} V_{\mu}^{A} V_{\nu}^{B}\right) A_{\rho}^{c} A_{\lambda}^{d}\right], \qquad (5.1)
\end{aligned}
$$

where we used the notations

$$
D^{\mu}\varphi_{a} = \partial^{\mu}\varphi_{a} + \lambda W_{ab} A^{b\mu} - \tfrac{\lambda}{4!} f_{Aa} V^{A\mu}, \qquad (5.2)
$$

$$
\begin{aligned}
\bar{F}_{\mu\nu}^{a} = & \partial_{[\mu} A_{\nu]}^{a} + \lambda M_{bc}^{a} A_{\mu}^{b} A_{\nu}^{c} + \lambda \varepsilon_{\mu\nu\rho\lambda} M^{ab} B_{b}^{\rho\lambda} \\
& + \tfrac{\lambda}{12}\left(f_{Ab}^{a} A_{[\mu}^{b} V_{\nu]}^{A} + g_{AB}^{a} V_{\mu}^{A} V_{\nu}^{B}\right), \qquad (5.3)
\end{aligned}
$$

$$
\bar{F}_{\mu\nu}^{A} = \partial_{[\mu} V_{\nu]}^{A} - \lambda f_{aB}^{A} A_{[\mu}^{a} V_{\nu]}^{B} + \lambda g_{BC}{}^{A} V_{\mu}^{B} V_{\nu}^{C}. \qquad (5.4)
$$

Formula (5.1) expresses the most general form of the Lagrangian action describing the interactions between a finite collection of BF models and a finite set of two-form gauge fields that complies with our working hypotheses and whose free limit is precisely action (2.1). We note that the deformed Lagrangian action is of maximum order 1 in the coupling constant and includes two main types of vertices: one generates self-interactions among the BF fields and the other couples the two-form field spectrum to the BF field spectrum. The first type is already known from the literature and we will not comment on it. The second is yielded by the expression

$$
\begin{aligned}
& -\tfrac{\lambda}{4!} f_{Aa} V^{A\mu} H_{\mu}^{a} + \tfrac{\lambda}{24} B_{a}^{\mu\nu}\left(f_{Ab}^{a} A_{[\mu}^{b} V_{\nu]}^{A} + g_{AB}^{a} V_{\mu}^{A} V_{\nu}^{B}\right) \\
& -\tfrac{\lambda}{2} V_{A}^{\mu\nu}\left(f_{aB}^{A} A_{[\mu}^{a} V_{\nu]}^{B} - g_{BC}{}^{A} V_{\mu}^{B} V_{\nu}^{C}\right) \\
& -\tfrac{\lambda}{4} \varepsilon^{\mu\nu\rho\lambda}\left(\tfrac{2}{3} f_{Aacd} V_{\mu}^{A} A_{\nu}^{a} - g_{ABcd} V_{\mu}^{A} V_{\nu}^{B}\right) A_{\rho}^{c} A_{\lambda}^{d}. \qquad (5.5)
\end{aligned}
$$

We observe that the vector fields $V^{A\mu}$ couple to all the BF fields from the collection, while the two-form gauge fields $V_A^{\mu\nu}$ interact only with the one-forms A_μ^a from the BF sector. Also, all the interaction vertices are derivative-free (we recall that the various functions that parameterize (5.1) depend only on the *undifferentiated* scalar fields). One of this couplings, $\frac{\lambda}{2}g_{BC}{}^{A}V_A^{\mu\nu}V_\mu^B V_\nu^C$, is nothing but the generalized version of non-Abelian Freedman-Townsend vertex. (By 'generalized' we mean that its form is identical with the standard non-Abelian Freedman-Townsend vertex up to the point that $g_{BC}{}^{A}$ are *not* the structure constants of a Lie algebra, but depend on the undifferentiated scalar fields.) Thus, action (5.1) contains the generalized version of non-Abelian Freedman-Townsend action

$$S_{\mathrm{gen}}^{\mathrm{FT}}[V_{\mu\nu}^A, V_\mu^A, \varphi_a] = \tfrac{1}{2}\int d^4x \left[V_A^{\mu\nu}\left(\partial_{[\mu}V_{\nu]}^A + \lambda g_{BC}{}^{A}V_\mu^B V_\nu^C\right) + V_\mu^A V_A^\mu\right]. \quad (5.6)$$

From the terms of antighost number 1 present in (4.111) we read the deformed gauge transformations (which leave invariant action (5.1)), namely

$$\bar{\delta}_\epsilon A_\mu^a = \left(D_\mu\right)^a{}_b\,\epsilon^b - 2\lambda M^{ab}\varepsilon_{\mu\nu\rho\lambda}\epsilon_b^{\nu\rho\lambda}, \quad (5.7)$$

$$\begin{aligned}
\bar{\delta}_\epsilon H_\mu^a \;=\;& 2\left(\bar{D}^\nu\right)^a{}_b\,\epsilon_{\mu\nu}^b + \tfrac{\lambda}{2}\varepsilon_{\mu\nu\rho\lambda}\left[\left(-\tfrac{1}{12}\frac{\partial M_{bcde}}{\partial\varphi_a}A^{c\nu} + \frac{\partial f_{bde}^A}{\partial\varphi_a}V_A^\nu\right)A^{d\rho}\right.\\
&\left.+\frac{\partial g_{be}^{AB}}{\partial\varphi_a}V_A^\nu V_B^\rho\right]A^{e\lambda}\epsilon^b + \lambda\left(-\frac{\partial W_{bc}}{\partial\varphi_a}H_\mu^c + \frac{\partial f_{bB}^A}{\partial\varphi_a}V_A^\nu V_{\mu\nu}^B\right)\epsilon^b\\
&-\frac{\partial\left(D^\nu\right)^d{}_b}{\partial\varphi_a}B_{d\mu\nu}\epsilon^b - \tfrac{3\lambda}{2}\frac{\partial M_{cd}^b}{\partial\varphi_a}A^{c\nu}A^{d\rho}\epsilon_{b\mu\nu\rho} + 2\lambda\frac{\partial M^{bc}}{\partial\varphi_a}B_{c\mu\nu}\varepsilon^{\nu\alpha\beta\gamma}\epsilon_{b\alpha\beta\gamma}\\
&+\tfrac{\lambda}{4}\left(\frac{\partial f_{Ac}^b}{\partial\varphi_a}V^{A\nu}A^{c\rho} - \tfrac{1}{2}\frac{\partial g_{AB}^b}{\partial\varphi_a}V^{A\nu}V^{B\rho}\right)\epsilon_{b\mu\nu\rho}\\
&+\lambda\varepsilon_{\mu\nu\rho\lambda}\left(\frac{\partial f_{bAB}}{\partial\varphi_a}V^{B\nu}A^{b\rho} + \tfrac{1}{2}\frac{\partial g^{BC}{}_A}{\partial\varphi_a}V_B^\nu V_C^\rho\right)\epsilon^{A\lambda}, \quad (5.8)
\end{aligned}$$

$$\bar{\delta}_\epsilon\varphi_a = -\lambda W_{ab}\epsilon^b, \quad (5.9)$$

$$\begin{aligned}
\bar{\delta}_\epsilon B_a^{\mu\nu} \;=\;& -3\left(D_\rho\right)_a{}^b\,\epsilon_b^{\mu\nu\rho} + 2\lambda W_{ab}\epsilon^{b\mu\nu} - \lambda\varepsilon^{\mu\nu\rho\lambda}f_{aAB}V_\rho^B\epsilon_\lambda^A - \lambda M_{ab}^c B_c^{\mu\nu}\epsilon^b\\
&+\lambda\varepsilon^{\mu\nu\rho\lambda}\left(\tfrac{1}{8}M_{abcd}A_\rho^c A_\lambda^d + f_{Aabc}V_\rho^A A_\lambda^c - \tfrac{1}{2}g_{ABab}V_\rho^A V_\lambda^B\right)\epsilon^b, \quad (5.10)
\end{aligned}$$

$$\bar{\delta}_\epsilon V_{\mu\nu}^A \;=\; \varepsilon_{\mu\nu\rho\lambda}\left(D^\rho\right)^A{}_B\,\epsilon^{B\lambda} + \tfrac{\lambda}{12}f_a^A\epsilon_{\mu\nu}^a + \tfrac{\lambda}{4}\left(f^{Aa}{}_b A^{b\rho} - g^{aAB}V_B^\rho\right)\epsilon_{a\mu\nu\rho}$$

$$+\lambda\left[\varepsilon_{\mu\nu\rho\lambda}\left(\tfrac{1}{2}f^{A}_{abc}A^{b\rho}+g^{AB}_{ac}V^{\rho}_{B}\right)A^{c\lambda}\right.$$
$$\left.+f^{A}_{aB}V^{B}_{\mu\nu}+\tfrac{1}{12}f^{Ab}_{\ \ a}B_{b\mu\nu}\right]\epsilon^{a}, \tag{5.11}$$

$$\bar{\delta}_{\epsilon}V^{A}_{\mu}=\lambda f^{A}_{aB}V^{B}_{\mu}\epsilon^{a}. \tag{5.12}$$

In (5.7)–(5.12) we employed the following notations for the various types of (generalized) covariant derivatives:

$$\left(\bar{D}^{\mu}\right)^{a}_{\ \ b} = \delta^{a}_{b}\partial^{\mu}-\lambda\left(\frac{\partial W_{bc}}{\partial\varphi_{a}}A^{c\mu}-\tfrac{1}{12}\frac{\partial f_{Ab}}{\partial\varphi_{a}}V^{A\mu}\right), \tag{5.13}$$
$$\left(D_{\mu}\right)^{a}_{\ \ b} = \delta^{a}_{b}\partial_{\mu}-\lambda M^{a}_{bc}A^{c}_{\mu}-\tfrac{\lambda}{12}f^{a}_{Ab}V^{A}_{\mu}, \tag{5.14}$$
$$\left(D_{\mu}\right)_{a}^{\ \ b} = \delta^{b}_{a}\partial_{\mu}+\lambda\left(M^{b}_{ac}A^{c}_{\mu}+\tfrac{1}{12}f^{b}_{Aa}V^{A}_{\mu}\right), \tag{5.15}$$
$$\left(D^{\mu}\right)^{A}_{\ \ B} = \delta^{A}_{B}\partial^{\mu}-\lambda f^{A}_{aB}A^{a\mu}+\lambda g^{AC}_{\ \ \ \ B}V^{\mu}_{C}. \tag{5.16}$$

It is interesting to see that the gauge transformations of all fields get modified by the deformation procedure. Also, the gauge transformations of the BF fields H^{a}_{μ} and $B^{\mu\nu}_{a}$ involve the gauge parameters $\epsilon^{A\lambda}$, which are specific to the two-form sector. Similarly, the gauge transformations of $V^{A}_{\mu\nu}$ and V^{A}_{μ} include pure BF gauge parameters. By contrast to the standard non-Abelian Freedman-Townsend model, where the vector fields V^{A}_{μ} are gauge-invariant, here these fields gain nonvanishing gauge transformations, proportional with the BF gauge parameters ϵ^{a}. The nonvanishing commutators among the deformed gauge transformations result from the terms quadratic in the ghosts with pure ghost number 1 present in (4.111). The concrete form of the gauge generators and of the corresponding nonvanishing commutators is included in Appendix B and D, respectively (see relations (B.1)–(B.16) and (D.1)–(D.19), respectively). With the help of these relations we observe that the original Abelian gauge algebra is deformed into an open one, meaning that the commutators among the gauge transformations only close on-shell, i.e. on the field equations resulting from the deformed Lagrangian action (5.1). The deformed gauge generators remain reducible of order two, just like the original ones, but the reducibility relations of order one and two hold now only on the field equations resulting from the deformed Lagrangian action (on-shell reducibility). The expressions of the reducibility functions and relations are given in detail in Appendix C (see formulas (C.1)–(C.26)). They are deduced from certain elements in (4.111) that are linear in the ghosts with the pure ghost number greater or equal to 2.

We recall that the entire gauge structure of the interacting model is controlled by the functions W_{ab}, M_{bc}^{a}, M_{abcd}, M^{ab}, f_{abc}^{A}, g_{ab}^{AB}, f_{a}^{A}, f_{aB}^{A}, $f^{Ab}_{\ \ a}$, $g^{AB}_{\quad C}$, and g^{aAB}, which are restricted to satisfy equations (4.105)–(4.108). Thus, our procedure is consistent provided these equations are shown to possess solutions. We give below some classes of solutions to (4.105)–(4.108), without pretending to exhaust all possibilities.

- **Type I solutions**

  A first class of solutions to equations (4.105) is given by

$$M_{ab}^{c}=\frac{\partial W_{ab}}{\partial \varphi_c},\quad M_{abcd}=f_{e[ab}\frac{\partial W_{cd]}}{\partial \varphi_e},\quad M^{ab}=0, \tag{5.17}$$

  where f_{eab} are arbitrary, antisymmetric constants and the functions W_{ab} are required to fulfill the equations

$$W_{e[a}\frac{\partial W_{bc]}}{\partial \varphi_e}=0. \tag{5.18}$$

  We remark that all the nonvanishing solutions are parameterized by the antisymmetric functions W_{ab}. Like in the pure BF case [51], we can interpret the functions W_{ab} like the components of a two-tensor on a Poisson manifold with the target space locally parameterized by the scalar fields φ_e. Consequently, the first and third equations among (4.106) are verified if we take

$$f_{aB}^{A}=\lambda^{A}_{\ B}f_a,\quad f_{a}^{A}=\tau^{A}k^{c}W_{ac},\quad f^{Aa}_{\ \ \ b}=-\tfrac{1}{2}\tau^{A}k^{c}\frac{\partial W_{bc}}{\partial \varphi_a}, \tag{5.19}$$

  where f_a are arbitrary functions of φ_b, k^c stand for some arbitrary constants, and τ^A and $\lambda^{A}_{\ B}$ ($\lambda^{AB}=-\lambda^{BA}$, $\lambda^{AB}=k^{AC}\lambda^{B}_{\ C}$) represent some constants subject to the conditions

$$\lambda^{A}_{\ B}\tau^{B}=0. \tag{5.20}$$

  Inserting (5.19) into the second equation from (4.106), we obtain

$$g_{AB}^{a}=\tfrac{1}{2}g_{ABC}\tau^{C}k^{a}+\mu_{AB}\nu^{a}, \tag{5.21}$$

  where μ_{AB} are some arbitrary, antisymmetric constants and $\nu^{a}(\varphi)$ are null vectors of W_{ab} (if the matrix of elements W_{ab} is degenerate), i.e.

$$W_{ab}\nu^{a}=0. \tag{5.22}$$

In the presence of the previous solutions the fourth equation from (4.106) is solved for

$$f^A_{abc} = \tfrac{1}{4 \cdot 4!} \tau^A k^d f_{e[ab} \frac{\partial W_{cd]}}{\partial \varphi_e}. \tag{5.23}$$

Due to the last relation in (5.17), it is easy to see that the fifth equation from (4.106) is now automatically satisfied. Next, we investigate equations (4.107). The former equation is checked if we make the choice

$$f_a = \bar{k}^b W_{ab}, \tag{5.24}$$

with $\bar{k}^b$ some arbitrary constants. The next equation from (4.107) is fulfilled for

$$g_{ABC} = C_{ABC}(1 + \chi), \quad \lambda^A{}_B = C_{CB}{}^A \tau^C, \quad k^a = \bar{k}^a, \tag{5.25}$$

where $\chi\left(\varphi\right)$ has the property

$$W_{ab} \frac{\partial \chi}{\partial \varphi_b} = 0 \tag{5.26}$$

(if W_{ab} allows for nontrivial null vectors) and the completely antisymmetric constants C_{ABC} are imposed to satisfy the Jacobi identity

$$C_{EA[B} C^E_{DC]} = 0. \tag{5.27}$$

Now, the third equation from (4.107) is automatically verified by the last relation in (5.17). The solution to the fourth equation reads as

$$g^{AB}_{ab} = C^{ABC} \tau_C W_{ab}, \quad \mu_{AB} = 0. \tag{5.28}$$

So far we have determined all the unknown functions. The above solutions also fulfill the remaining equations from (4.107) and the first three ones in (4.108). However, the last equation present in (4.108) produces the restriction

$$C^{E[AB} C^{CD]F} \tau_E \tau_F = 0. \tag{5.29}$$

The last equation possesses at least two different types of solutions, namely

$$C^{ABC} = \varepsilon^{ijk} e^A_i e^B_j e^C_k, \quad i, j, k = 1, 2, 3 \tag{5.30}$$

and

$$C^{ABC} = \varepsilon^{\bar{A}\bar{B}\bar{C}} l^A_{\bar{A}} l^B_{\bar{B}} l^C_{\bar{C}}, \quad \bar{A}, \bar{B}, \bar{C} = 1, 2, 3, 4, \tag{5.31}$$

respectively, where e^A_i and $l^A_{\bar{A}}$ are all constants and ε^{ijk} together with $\varepsilon^{\bar{A}\bar{B}\bar{C}}$ are completely antisymmetric symbols. These symbols are defined via the conventions $\varepsilon^{123} = +1$ and $\varepsilon^{124} = \varepsilon^{134} = \varepsilon^{234} = +1$, respectively. It is straightforward to see that the quantities C^{ABC} given by either of the relations (5.30) or (5.31) indeed check (5.27). By assembling the previous results, we find the type I solutions to equations (4.105)–(4.108) being expressed via relations (5.17), (5.23), and

$$f^A_{aB} = C_{DB}{}^A \tau^D k^b W_{ab}, \quad f^A_a = \tau^A k^c W_{ac}, \tag{5.32}$$

$$f^{Aa}_{\ \ b} = -\tfrac{1}{2} \tau^A k^c \frac{\partial W_{bc}}{\partial \varphi_a}, \quad g_{ABC} = C_{ABC}(1 + \chi), \tag{5.33}$$

$$g^a_{AB} = \tfrac{1}{2} C_{ABC}(1 + \chi) \tau^C k^a, \quad g^{AB}_{ab} = C^{ABC} \tau_C W_{ab}, \tag{5.34}$$

where τ^A and k^a represent some arbitrary constants, W_{ab} are assumed to satisfy equations (5.18), and χ is subject to (5.26) (if the matrix of elements W_{ab} is degenerate). The antisymmetric constants C^{ABC} are imposed to verify relations (5.29) (which ensure that (5.27) are automatically checked). Two sets of solutions to (5.29) (and hence also to (5.27)) are provided by formulas (5.30) and (5.31)).

- **Type II solutions**

  Another set of solutions to equations (4.105) can be written as

$$W_{ab} = 0, \quad M^c_{ab} = C^c{}_{ab} \hat{M}, \quad M_{abcd} = 0, \quad M^{ab} = \mu^{ab} M, \tag{5.35}$$

with $\hat{M}$ and M arbitrary functions of the undifferentiated scalar fields. The coefficients μ^{ab} represent the elements of the inverse of the Killing metric $\bar{\mu}_{ad}$ of a semi-simple Lie algebra with the structure constants $C^c{}_{ab}$ ($\bar{\mu}_{ad} \mu^{de} = \delta^e_a$), where, in addition $C_{abc} = \bar{\mu}_{ad} C^d{}_{bc}$ must be completely antisymmetric. Under these circumstances, the first equation from (4.106) is solved if we take

$$f^A_{aB} = \tilde{\lambda}^A{}_B \hat{f}_a, \quad f^A_a = \sigma^A \bar{f}_a, \tag{5.36}$$

where $\hat{f}_a$ and $\bar{f}_a$ are arbitrary functions of the undifferentiated scalar fields, and $\tilde{\lambda}^A{}_B$ as well as σ^A are some constants that must satisfy the relations

$$\tilde{\lambda}^A{}_B \sigma^B = 0. \tag{5.37}$$

Then, the second equation from (4.106) implies the fact that ${g_{AB}}^{C}$ is restricted to fulfill the condition

$${g_{AB}}^{C}\sigma_C = 0. \tag{5.38}$$

Replacing the above solutions into the third equation from (4.106), we get the relation

$$f^{Aa}_{\ \ b} = \sigma^A C^a_{\ bc}\frac{\partial P}{\partial \varphi_c}, \quad f^A_{abc} = \sigma^A C_{abc} N, \tag{5.39}$$

where P and N are functions of the undifferentiated scalar fields, with N restricted to verify the equation

$$\bar{f}_a \frac{\partial \hat{M}}{\partial \varphi_a} + 4 \cdot 4! N M = 0. \tag{5.40}$$

Having in mind the solutions deduced until now, we find that the fourth equation from (4.106) is automatically checked and the last equation in (4.106) constrains the function M to be constant (for the sake of simplicity, we take this constant to be equal to unity)

$$M = 1. \tag{5.41}$$

The first and the third equations from (4.107) immediately yield $\hat{f}_a = 0$, which further leads to $f^A_{aB} = 0$. Under these circumstances, the second equation entering (4.107) is identically satisfied and the fourth equation from the same formula possesses the solution

$$g^{AB}_{ab} = C_{abc}\bar{\lambda}^{AB}\frac{\partial Q}{\partial \varphi_c}, \tag{5.42}$$

where Q is an arbitrary function of the undifferentiated scalar fields and $\bar{\lambda}^{AB}$ denote some arbitrary, completely antisymmetric constants. Substituting the solutions deduced so far into the last equation from (4.107), we get

$$g^a_{AB} = \bar{\lambda}_{AB}\frac{\partial g}{\partial \varphi_a}, \tag{5.43}$$

where g is a function of the undifferentiated scalar fields that is restricted to fulfill the equation

$$\frac{\partial Q}{\partial \varphi_a} = \tfrac{1}{2 \cdot 4!}\hat{M}\frac{\partial g}{\partial \varphi_a}. \tag{5.44}$$

The first equation from (4.108) exhibits the solution

$$g_{ABC} = \sigma_{[A} \hat{\lambda}_{B]C} \hat{\Phi}, \tag{5.45}$$

with $\hat{\Phi}$ an arbitrary function of the undifferentiated scalar fields and $\hat{\lambda}_{BC}$ some arbitrary, completely antisymmetric constants, which check the relations

$$\hat{\lambda}_{BC} \sigma^{C} = 0. \tag{5.46}$$

Relations (5.46) ensure that equation (5.38) is verified. The second equation from (4.108) displays a solution of the form

$$\bar{\lambda}^{AB} = \sigma^{[A} \hat{\lambda}^{B]C} \beta_{C}, \tag{5.47}$$

with β_C some constants. The remaining equations entering (4.108) are now identically verified. Putting together the results obtained until now, it follows that the type II solutions to equations (4.105)–(4.108) can be written as

$$W_{ab} = 0, \quad M_{ab}^{c} = C^{c}{}_{ab} \hat{M}, \quad M_{abcd} = 0, \quad M^{ab} = \mu^{ab}, \tag{5.48}$$

$$f_{aB}^{A} = 0, \quad f_{a}^{A} = \sigma^{A} \bar{f}_{a}, \quad f_{\ b}^{Aa} = \sigma^{A} C^{a}{}_{bc} \frac{\partial P}{\partial \varphi_{c}}, \tag{5.49}$$

$$f_{abc}^{A} = -\tfrac{1}{4 \cdot 4!} \sigma^{A} C_{abc} \bar{f}_{d} \frac{\partial \hat{M}}{\partial \varphi_{d}}, \quad g_{ab}^{AB} = \tfrac{1}{2 \cdot 4!} C_{abc} \sigma^{[A} \hat{\lambda}^{B]C} \beta_{C} \hat{M} \frac{\partial g}{\partial \varphi_{c}}, \tag{5.50}$$

$$g_{AB}^{a} = \sigma_{[A} \hat{\lambda}_{B]C} \beta^{C} \frac{\partial g}{\partial \varphi_{a}}, \quad g_{ABC} = \sigma_{[A} \hat{\lambda}_{B]C} \hat{\Phi}. \tag{5.51}$$

We recall that $\hat{M}$, $\bar{f}_a$, P, g, and $\hat{\Phi}$ are arbitrary functions of the undifferentiated scalar fields and β_C, $\hat{\lambda}_{BC}$, and σ^C are some constants. In addition, the last two sets of constants are imposed to fulfill equation (5.46). The quantities μ^{ab} are the elements of the inverse of the Killing metric of a semi-simple Lie algebra with the structure constants $C^{c}{}_{ab}$, where C_{abc} must be completely antisymmetric.

- **Type III solutions**

The third type of solutions to (4.105) is given by

$$W_{ab} = 0, \quad M_{ab}^{c} = \bar{C}^{c}{}_{ab} w, \quad M_{abcd} = \hat{f}_{e[ab} \bar{C}^{e}{}_{cd]} q, \quad M^{ab} = 0, \tag{5.52}$$

with w and q arbitrary functions of the undifferentiated scalar fields, $\hat{f}_{eab}$ some arbitrary, antisymmetric constants, and $\bar{C}^{c}{}_{ab}$ the structure constants of a Lie algebra. Let us particularize the last solutions to the case where

$$\bar{C}^{c}{}_{ab}=\hat{k}^{c}\bar{W}_{ab},\quad w\left(\varphi\right)=q\left(\varphi\right)=\frac{d\hat{w}\left(\hat{k}^{m}\varphi_{m}\right)}{d\left(\hat{k}^{n}\varphi_{n}\right)},\tag{5.53}$$

with $\hat{k}^{c}$ some arbitrary constants, $\hat{w}$ an arbitrary, smooth function depending on $\hat{k}^{m}\varphi_{m}$, and $\bar{W}_{ab}$ some antisymmetric constants satisfying the relations

$$\bar{W}_{a[b}\bar{W}_{cd]}=0.\tag{5.54}$$

Obviously, equations (5.54) ensure the Jacobi identity for the structure constants $\bar{C}^{c}{}_{ab}$. Replacing (5.53) back in (5.52), we find

$$W_{ab}=0,\quad M_{ab}^{c}=\frac{\partial\hat{W}_{ab}}{\partial\varphi_{c}},\quad M_{abcd}=\hat{f}_{e[ab}\frac{\partial\hat{W}_{cd]}}{\partial\varphi_{e}},\quad M^{ab}=0,\tag{5.55}$$

where

$$\hat{W}_{ab}=\bar{W}_{ab}\frac{d\hat{w}\left(\hat{k}^{m}\varphi_{m}\right)}{d\left(\hat{k}^{n}\varphi_{n}\right)}.\tag{5.56}$$

Due to (5.54), it is easy to see that $\hat{W}_{ab}$ satisfy the Jacobi identity for a Poisson manifold

$$\hat{W}_{e[a}\frac{\partial\hat{W}_{bc]}}{\partial\varphi_{e}}=0.\tag{5.57}$$

Relations (5.55) and (5.57) emphasize that we can generate solutions correlated with a Poisson manifold even if $W_{ab}=0$. In this situation the Poisson two-tensor results from a Lie algebra (see the first formula in (5.53) and (5.56)). It is interesting to remark that the same equations, namely (5.54), ensure the Jacobi identities for both the Lie algebra and the corresponding Poisson manifold. These equations possess at least two types of solutions, namely

$$\bar{W}_{ab}=\varepsilon_{ijk}e_{a}^{i}e_{b}^{j}e_{c}^{k}\rho^{c},\quad i,j,k=1,2,3\tag{5.58}$$

and

$$\bar{W}_{ab}=\varepsilon_{\bar{a}\bar{b}\bar{c}}l_{a}^{\bar{a}}l_{b}^{\bar{b}}l_{c}^{\bar{c}}\bar{\rho}^{c},\quad \bar{a},\bar{b},\bar{c}=1,2,3,4,\tag{5.59}$$

where e^i_a, ρ^c, $l^{\bar{a}}_a$, and $\bar{\rho}^c$ are all constants and ε_{ijk} together with $\varepsilon_{\bar{a}\bar{b}\bar{c}}$ are completely antisymmetric symbols, defined via the conventions $\varepsilon_{123} = +1$ and $\varepsilon_{124} = \varepsilon_{134} = \varepsilon_{234} = +1$, respectively. If we tackle the remaining equations in a manner similar to that employed at the previous cases, we infer that the third type of solutions to (4.105)–(4.108) is expressed by (5.55) and

$$f^A_{aB} = m^A{}_B \hat{k}^b \bar{W}_{ab} \Omega, \quad f^A_a = 0, \quad f^{Aa}_{\ \ b} = -\bar{\lambda}^A \tilde{k}^c \frac{\partial \hat{W}_{bc}}{\partial \varphi_a}, \tag{5.60}$$

$$f^A_{abc} = \bar{\lambda}^A \left(\hat{u}_{[a} \hat{W}_{bc]} + \tfrac{1}{2\cdot 4!} \tilde{k}^d \hat{f}_{e[ab} \frac{\partial \hat{W}_{cd]}}{\partial \varphi_e} \right), \tag{5.61}$$

$$g^{AB}_{ab} = \bar{\lambda}^{[A} m^{B]C} \bar{\beta}_C \bar{W}_{ab} \hat{Q}, \quad g^a_{AB} = 0, \quad g_{ABC} = \bar{\lambda}_{[A} m_{B]C} \hat{P}. \tag{5.62}$$

In the above $\hat{k}^b$, $\tilde{k}^a$, $\bar{\beta}_C$, $\hat{f}_{eab}$, $\bar{\lambda}^A$, $\bar{W}_{ab}$ ($\bar{W}_{ab} = -\bar{W}_{ba}$), and m^{AB} ($m^{AB} = -m^{BA}$) are some constants, the first four sets being arbitrary (up to the point that $\hat{f}_{eab}$ should be completely antisymmetric) and the last three sets being subject to the relations (5.54) and

$$m^{AB} \bar{\lambda}_B = 0. \tag{5.63}$$

The quantities denoted by Ω, $\hat{u}_a$, $\hat{Q}$, and $\hat{P}$ are arbitrary functions of the undifferentiated scalar fields. The functions $\hat{W}_{ab}$ read as in (5.56), with $\hat{w}$ an arbitrary, smooth function depending on $\hat{k}^m \varphi_m$. If in particular we take Ω and $\hat{Q}$ to be respectively of the form of w and q from (5.53), then we obtain that the functions f^A_{aB} and g^{AB}_{ab} will be parameterized by $\hat{W}_{ab}$.

6 Conclusion

To conclude with, in this paper we have investigated the consistent interactions that can be introduced between a finite collection of BF theories and a finite set of two-form gauge fields (described by a sum of Abelian Freedman-Townsend actions). Starting with the BRST differential for the free theory, we compute the consistent first-order deformation of the solution to the master equation with the help of standard cohomological techniques, and obtain that it is parameterized by 11 kinds of functions depending on

the undifferentiated scalar fields. Next, we investigate the second-order deformation, whose existence imposes certain restrictions with respect to these functions. Based on these restrictions, we show that we can take all the remaining higher-order deformations to vanish. As a consequence of our procedure, we are led to an interacting gauge theory with deformed gauge transformations, a non-Abelian gauge algebra that only closes on-shell, and on-shell accompanying reducibility relations. The deformed action contains, among others, the generalized version of non-Abelian Freedman-Townsend action. It is interesting to mention that by contrast to the standard non-Abelian Freedman-Townsend model, where the auxiliary vector fields are gauge-invariant, here these fields gain nonvanishing gauge transformations, proportional with some BF gauge parameters. Finally, we investigate the equations that restrict the functions parameterizing the deformed solution to the master equation and give some particular classes of solutions, which can be suggestively interpreted in terms of Poisson manifolds and/or Lie algebras.

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A Various notations used in subsection 4.3

The various notations used within formula (4.84) are listed below. The objects denoted by $\left(K^{abc}_{,m_1\ldots m_p}\right)_{p=\overline{0,4}}$ are expressed by

$$\begin{aligned} K^{abc} = {} & \eta^a\eta^b\varphi^{*c} + 2\eta^a A^{b\mu} H^c_\mu + 2\left(A^{a\mu}A^{b\nu} - 2B^{*a\mu\nu}\eta^b\right) C^c_{\mu\nu} \\ & +4\left(\eta^a\eta^{*b\mu\nu\rho} + 3B^{*a\mu\nu}A^{b\rho}\right) C^c_{\mu\nu\rho} \\ & -4\left(\eta^a\eta^{*b\mu\nu\rho\lambda} + 6B^{*a\mu\nu}B^{*b\rho\lambda} - 4\eta^{*a\mu\nu\rho}A^{b\lambda}\right) C^c_{\mu\nu\rho\lambda}, \end{aligned} \tag{A.1}$$

$$\begin{aligned} K^{abc}_{,d} = {} & \left(4H^{*\nu}_d A^{a\mu}\eta^b - C^{*\mu\nu}_d \eta^a\eta^b\right) C^c_{\mu\nu} - H^{*\mu}_d \eta^a\eta^b H^c_\mu \\ & + \left(6H^{*\rho}_d A^{a\mu}A^{b\nu} - 12H^{*\rho}_d B^{*a\mu\nu}\eta^b + 6C^{*\mu\nu}_d \eta^a A^{b\rho}\right. \end{aligned}$$

$$
\begin{aligned}
&-C_d^{*\mu\nu\rho}\eta^a\eta^b\big)\, C^c_{\mu\nu\rho} + \big(-48H_d^{*\lambda}B^{*a\mu\nu}A^{b\rho} \\
&+12C_d^{*\mu\nu}A^{a\rho}A^{b\lambda} + 16H_d^{*\lambda}\eta^{*a\mu\nu\rho}\eta^b - 24C_d^{*\mu\nu}B^{*a\rho\lambda}\eta^b \\
&-8C_d^{*\mu\nu\rho}A^{a\lambda}\eta^b - C_d^{*\mu\nu\rho\lambda}\eta^a\eta^b\Big)\, C^c_{\mu\nu\rho\lambda},
\end{aligned}
\tag{A.2}
$$

$$
\begin{aligned}
K^{abc}_{,de} \;=\;& -3\left(C_d^{*\mu\nu}H_e^{*\rho}\eta^a + 2H_d^{*\mu}H_e^{*\nu}A^{a\rho}\right)\eta^b C^c_{\mu\nu\rho} \\
& -H_d^{*\mu}H_e^{*\nu}\eta^a\eta^b C^c_{\mu\nu} + \big(-24H_d^{*\mu}H_e^{*\nu}B^{*a\rho\lambda}\eta^b \\
& +12H_d^{*\mu}H_e^{*\nu}A^{a\rho}A^{b\lambda} - 24C_d^{*\mu\nu}H_e^{*\rho}A^{a\lambda}\eta^b \\
& -3C_d^{*\mu\nu}C_e^{*\rho\lambda}\eta^a\eta^b + 4C_d^{*\mu\nu\rho}H_e^{*\lambda}\eta^a\eta^b\big)\, C^c_{\mu\nu\rho\lambda},
\end{aligned}
\tag{A.3}
$$

$$
\begin{aligned}
K^{abc}_{,def} \;=\;& -2\left(4H_d^{*\mu}H_e^{*\nu}H_f^{*\rho}A^{a\lambda} + 3C_d^{*\mu\nu}H_e^{*\rho}H_f^{*\lambda}\eta^a\right)\eta^b C^c_{\mu\nu\rho\lambda} \\
& -H_d^{*\mu}H_e^{*\nu}H_f^{*\rho}\eta^a\eta^b C^c_{\mu\nu\rho},
\end{aligned}
\tag{A.4}
$$

$$
K^{abc}_{,defg} = -H_d^{*\mu}H_e^{*\nu}H_f^{*\rho}H_g^{*\lambda}\eta^a\eta^b C^c_{\mu\nu\rho\lambda}. \tag{A.5}
$$

The elements $\left(K^{abc}_{d,m_1\ldots m_p}\right)_{p=\overline{0,4}}$ read as

$$
\begin{aligned}
K^{abc}_d \;=\;& \left(-2\eta^a A^b_\mu A^c_\nu + B^{*a}_{\mu\nu}\eta^b\eta^c\right) B_d^{\mu\nu} - A^a_\mu\eta^b\eta^c A_d^{*\mu} \\
& + \left(-A^a_\mu A^b_\nu A^c_\rho + 6\eta^a B^{*b}_{\mu\nu}A^c_\rho + \eta^b\eta^c\eta^{*a}_{\mu\nu\rho}\right)\eta_d^{\mu\nu\rho} \\
& -\tfrac{1}{3}\eta^a\eta^b\eta^c\eta_d^* + \big(-12A^a_\mu A^b_\nu B^{*c}_{\rho\lambda} + 12\eta^a B^{*b}_{\mu\nu}B^{*c}_{\rho\lambda} \\
& -8\eta^a\eta^{*b}_{\mu\nu\rho}A^c_\lambda + \eta^{*c}_{\mu\nu\rho\lambda}\eta^a\eta^b\big)\,\eta_d^{\mu\nu\rho\lambda},
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
K^{abc}_{d,e} \;=\;& \left(H_e^{*\mu}A^{a\nu}\eta^b\eta^c + \tfrac{1}{6}C_e^{*\mu\nu}\eta^a\eta^b\eta^c\right) B_{d\mu\nu} \\
& -\tfrac{1}{3}H_e^{*\mu}\eta^a\eta^b\eta^c A^*_{d\mu} + \big(-3H_e^{*\rho}\eta^a A^{b\mu}A^{c\nu} \\
& -3H_e^{*\rho}\eta^a\eta^b B^{*c\mu\nu} + \tfrac{3}{2}C_e^{*\mu\nu}\eta^a\eta^b A^{c\rho} \\
& +\tfrac{1}{6}C_e^{*\mu\nu\rho}\eta^a\eta^b\eta^c\big)\,\eta_{d\mu\nu\rho} + \big(24A^{a\mu}H_e^{*\nu}\eta^b B^{*c\rho\lambda} \\
& +4H_e^{*\lambda}A^{a\mu}A^{b\nu}A^{c\rho} - 4H_e^{*\lambda}\eta^a\eta^b\eta^{*c\mu\nu\rho} \\
& +6C_e^{*\mu\nu}\eta^a\eta^b B^{*c\rho\lambda} - 6C_e^{*\mu\nu}\eta^a A^{b\rho}A^{c\lambda} \\
& +8C_e^{*\mu\nu\rho}\eta^a\eta^b A^{c\lambda} + \tfrac{1}{6}C_e^{*\mu\nu\rho\lambda}\eta^a\eta^b\eta^c\big)\,\eta_{d\mu\nu\rho\lambda},
\end{aligned}
\tag{A.7}
$$

$$
K^{abc}_{d,ef} \;=\; \tfrac{1}{6}H_e^{*\mu}H_f^{*\nu}\eta^a\eta^b\eta^c B_{d\mu\nu} + \tfrac{3}{2}H_e^{*\mu}H_f^{*\nu}\eta^a\eta^b A^{c\rho}\eta_{d\mu\nu\rho}
$$

$$
\begin{aligned}
&+\tfrac{1}{2}C_e^{*\mu\nu}H_f^{*\rho}\eta^a\eta^b\eta^c\eta_{d\mu\nu\rho}+\left(6H_e^{*\mu}H_f^{*\nu}\eta^a\eta^b B^{*c\rho\lambda}\right.\\
&-6H_e^{*\mu}H_f^{*\nu}\eta^a A^{b\rho}A^{c\lambda}+6C_e^{*\mu\nu}H_f^{*\rho}\eta^a\eta^b A^{c\lambda}\\
&\left.+\tfrac{2}{3}C_e^{*\mu\nu\rho}H_f^{*\lambda}\eta^a\eta^b\eta^c+\tfrac{1}{2}C_e^{*\mu\nu}C_f^{*\rho\lambda}\eta^a\eta^b\eta^c\right)\eta_{d\mu\nu\rho\lambda}, \qquad \text{(A.8)}
\end{aligned}
$$

$$
\begin{aligned}
K_{d,efg}^{abc} &= \left(2H_e^{*\mu}H_f^{*\nu}H_g^{*\rho}\eta^a\eta^b A^{c\lambda}+C_e^{*\mu\nu}H_f^{*\rho}H_g^{*\lambda}\eta^a\eta^b\eta^c\right)\eta_{d\mu\nu\rho\lambda}\\
&\quad+\tfrac{1}{6}H_e^{*\mu}H_f^{*\nu}H_g^{*\rho}\eta^a\eta^b\eta^c\eta_{d\mu\nu\rho}, \qquad \text{(A.9)}
\end{aligned}
$$

$$
K_{d,efgh}^{abc}=\tfrac{1}{6}H_e^{*\mu}H_f^{*\nu}H_g^{*\rho}H_h^{*\lambda}\eta^a\eta^b\eta^c\eta_{d\mu\nu\rho\lambda}. \qquad \text{(A.10)}
$$

The quantities $\left(K_{m_1\ldots m_p}^{abcdf}\right)_{p=\overline{0,4}}$, $\left(K_{b,m_1\ldots m_p}^{a}\right)_{p=\overline{0,4}}$, and $\left(K_{ab,m_1\ldots m_p}^{c}\right)_{p=\overline{0,4}}$ are given by

$$
\begin{aligned}
K^{abcdf} &= \tfrac{1}{8}\varepsilon^{\mu\nu\rho\lambda}\left[\left(\tfrac{1}{3!}A_\mu^a A_\nu^b-B_{\mu\nu}^{*a}\eta^b\right)A_\rho^c A_\lambda^d+\tfrac{1}{3}\left(B_{\mu\nu}^{*a}B_{\rho\lambda}^{*b}\right.\right.\\
&\quad\left.\left.-\tfrac{2}{3}\eta_{\mu\nu\rho}^{*a}A_\lambda^b+\tfrac{1}{4!}\eta_{\mu\nu\rho\lambda}^{*a}\eta^b\right)\eta^c\eta^d\right]\eta^f, \qquad \text{(A.11)}
\end{aligned}
$$

$$
\begin{aligned}
K_e^{abcdf} &= \tfrac{1}{4!}\varepsilon^{\mu\nu\rho\lambda}\left[\tfrac{1}{2}\left(\tfrac{1}{5!}C_{e\mu\nu\rho\lambda}^{*}\eta^a+\tfrac{1}{3!}C_{e\mu\nu\rho}^{*}A_\lambda^a+\tfrac{1}{2}C_{e\mu\nu}^{*}B_{\rho\lambda}^{*a}\right.\right.\\
&\quad\left.+\tfrac{1}{3}H_{e\mu}^{*}\eta_{\nu\rho\lambda}^{*a}\right)\eta^b\eta^c-H_{e\mu}^{*}\left(A_\nu^a A_\rho^b-2B_{\nu\rho}^{*a}\eta^b\right)A_\lambda^c\\
&\quad\left.-\tfrac{1}{2}C_{e\mu\nu}^{*}A_\rho^a A_\lambda^b\eta^c\right]\eta^d\eta^f, \qquad \text{(A.12)}
\end{aligned}
$$

$$
\begin{aligned}
K_{eg}^{abcdf} &= \tfrac{1}{2\cdot 4!}\varepsilon^{\mu\nu\rho\lambda}\left[\tfrac{1}{2}\left(\tfrac{1}{15}H_{e\mu}^{*}C_{g\nu\rho\lambda}^{*}\eta^a+\tfrac{1}{20}C_{e\mu\nu}^{*}C_{g\rho\lambda}^{*}\eta^a+H_{e\mu}^{*}C_{g\nu\rho}^{*}A_\lambda^a\right)\eta^b\right.\\
&\quad\left.-H_{e\mu}^{*}H_{g\nu}^{*}\left(A_\rho^a A_\lambda^b-2B_{\rho\lambda}^{*a}\eta^b\right)\right]\eta^c\eta^d\eta^f, \qquad \text{(A.13)}
\end{aligned}
$$

$$
K_{egh}^{abcdf}=\tfrac{1}{4\cdot 4!}\varepsilon^{\mu\nu\rho\lambda}H_{e\mu}^{*}H_{g\nu}^{*}\left(\tfrac{1}{10}C_{h\rho\lambda}^{*}\eta^a+\tfrac{1}{3}H_{h\rho}^{*}A_\lambda^a\right)\eta^b\eta^c\eta^d\eta^f, \qquad \text{(A.14)}
$$

$$
K_{eghl}^{abcdf}=\tfrac{1}{2\cdot 4!\cdot 5!}\varepsilon^{\mu\nu\rho\lambda}H_{e\mu}^{*}H_{g\nu}^{*}H_{h\rho}^{*}H_{l\lambda}^{*}\eta^a\eta^b\eta^c\eta^d\eta^f, \qquad \text{(A.15)}
$$

$$
\begin{aligned}
K_b^a &= 4\varepsilon^{\mu\nu\rho\lambda}\left[2\left(-C_{\mu\nu\rho\lambda}^{a}\eta_b^{*}+C_{\mu\nu\rho}^{a}A_{b\lambda}^{*}\right)+C_{\mu\nu}^{a}B_{b\rho\lambda}\right.\\
&\quad\left.-\left(\varphi^{*a}\eta_{b\mu\nu\rho\lambda}-H_\mu^a\eta_{b\nu\rho\lambda}\right)\right], \qquad \text{(A.16)}
\end{aligned}
$$

$$
\begin{aligned}
K_{b,c}^a &= 4\varepsilon^{\mu\nu\rho\lambda}\left[\eta_{b\mu\nu\rho\lambda}\left(C_{\sigma\tau\kappa\varsigma}^{a}C_c^{*\sigma\tau\kappa\varsigma}+C_{\sigma\tau\kappa}^{a}C_c^{*\sigma\tau\kappa}+C_{\sigma\tau}^{a}C_c^{*\sigma\tau}\right.\right.\\
&\quad\left.+H_\sigma^a H_c^{*\sigma}\right)+C_{\mu\nu\rho\lambda}^{a}\left(\eta_{b\sigma\tau\kappa}C_c^{*\sigma\tau\kappa}+B_{b\sigma\tau}C_c^{*\sigma\tau}-2A_{b\sigma}^{*}H_c^{*\sigma}\right)\\
&\quad\left.+\eta_{b\nu\rho\lambda}\left(3C_{\mu\sigma\tau}^{a}C_c^{*\sigma\tau}-2C_{\mu\sigma}^{a}H_c^{*\sigma}\right)+3B_{b\rho\lambda}C_{\mu\nu\sigma}^{a}H_c^{*\sigma}\right], \qquad \text{(A.17)}
\end{aligned}
$$

$$
\begin{aligned}
K^a_{b,cd} &= 4\varepsilon^{\mu\nu\rho\lambda}\Big[\eta_{b\mu\nu\rho\lambda}\big(C^a_{\sigma\tau\kappa\varsigma}\left(4H^{*\sigma}_c C^{*\tau\kappa\varsigma}_d + 3C^{*\sigma\tau}_c C^{*\kappa\varsigma}_d\right) \\
&\quad +3C^a_{\sigma\tau\kappa}H^{*\sigma}_c C^{*\tau\kappa}_d + C^a_{\sigma\tau}H^{*\sigma}_c H^{*\tau}_d\big) \\
&\quad +C^a_{\mu\nu\rho\lambda}\left(3\eta_{b\sigma\tau\kappa}H^{*\sigma}_c C^{*\tau\kappa}_d + B_{b\sigma\tau}H^{*\sigma}_c H^{*\tau}_d\right)\Big],
\end{aligned} \tag{A.18}
$$

$$
\begin{aligned}
K^a_{b,cde} &= 4\varepsilon^{\mu\nu\rho\lambda}\Big[\eta_{b\mu\nu\rho\lambda}\left(6C^a_{\sigma\tau\kappa\varsigma}H^{*\sigma}_c H^{*\tau}_d C^{*\kappa\varsigma}_e + C^a_{\sigma\tau\kappa}H^{*\sigma}_c H^{*\tau}_d H^{*\kappa}_e\right) \\
&\quad +C^a_{\mu\nu\rho\lambda}\eta_{b\sigma\tau\kappa}H^{*\sigma}_c H^{*\tau}_d H^{*\kappa}_e\Big],
\end{aligned} \tag{A.19}
$$

$$
K^a_{b,cdef} = 4\varepsilon_{\mu\nu\rho\lambda}\eta^{\mu\nu\rho\lambda}_b C^a_{\sigma\tau\kappa\varsigma}H^{*\sigma}_c H^{*\tau}_d H^{*\kappa}_e H^{*\varsigma}_f, \tag{A.20}
$$

$$
\begin{aligned}
K^c_{ab} &= \varepsilon_{\mu\nu\rho\lambda}\Big[-6\left(\eta^{\mu\nu\sigma}_a B^{\rho\lambda}_b A^c_\sigma + 3\eta^{\mu\sigma\tau}_a \eta^\nu_{b\sigma\tau}B^{*c\rho\lambda}\right) \\
&\quad -2\eta^{\mu\nu\rho\lambda}_a\left(\eta^{\sigma\tau\kappa\varsigma}_b \eta^{*c}_{\sigma\tau\kappa\varsigma} + 2\eta^{\sigma\tau\kappa}_b \eta^{*c}_{\sigma\tau\kappa} + 2B^{\sigma\tau}_b B^{*c}_{\sigma\tau}\right. \\
&\quad \left.-2A^{*\sigma}_b A^c_\sigma - 2\eta^*_b\eta^c\right) + 4\eta^{\mu\nu\rho}_a A^{*\lambda}_b \eta^c - B^{\mu\nu}_a B^{\rho\lambda}_b \eta^c\Big],
\end{aligned} \tag{A.21}
$$

$$
\begin{aligned}
K^c_{ab,d} &= \varepsilon_{\mu\nu\rho\lambda}\Big[-9\eta^{\mu\sigma\tau}_a \eta^\nu_{b\sigma\tau}\left(\eta^c C^{*\rho\lambda}_d - 2A^{c\rho}H^{*\lambda}_d\right) \\
&\quad -\eta^{\sigma\tau\kappa\varsigma}_a \eta_{b\sigma\tau\kappa\varsigma}\Big(\eta^c C^{*\mu\nu\rho\lambda}_d + 4C^{*\mu\nu\rho}_d A^{c\lambda} \\
&\quad +12C^{*\mu\nu}_d B^{*c\rho\lambda} + 8H^{*\mu}_d \eta^{*c\nu\rho\lambda}\Big) + 6\eta^{\mu\nu\sigma}_a B^{\rho\lambda}_b \eta^c H^*_{d\sigma} \\
&\quad -2\eta^{\mu\nu\rho\lambda}_a\left(\eta^{\sigma\tau\kappa}_b\left(\eta^c C^*_{d\sigma\tau\kappa} + 3A^c_\kappa C^*_{d\sigma\tau} - 6B^{*c}_{\tau\kappa}H^*_{d\sigma}\right)\right. \\
&\quad \left.+2A^{*\sigma}_b \eta^c H^*_{d\sigma} + B^{\sigma\tau}_b\left(\eta^c C^*_{d\sigma\tau} + 2A^c_\tau H^*_{d\sigma}\right)\right)\Big],
\end{aligned} \tag{A.22}
$$

$$
\begin{aligned}
K^c_{ab,de} &= -\varepsilon_{\mu\nu\rho\lambda}\Big[2\eta^{\mu\nu\rho\lambda}_a\left(3\eta^{\sigma\tau\kappa}_b\left(H^*_{d\sigma}C^*_{e\tau\kappa}\eta^c + H^*_{d\sigma}H^*_{e\tau}A^c_\kappa\right)\right. \\
&\quad \left.+B^{\sigma\tau}_b H^*_{d\sigma}H^*_{e\tau}\eta^c\right) + \eta^{\sigma\tau\kappa\varsigma}_a \eta_{b\sigma\tau\kappa\varsigma}\Big(\big(4H^{*\mu}_d C^{*\nu\rho\lambda}_e \\
&\quad +3C^{*\mu\nu}_d C^{*\rho\lambda}_e\big)\eta^c + 12H^{*\mu}_d C^{*\nu\rho}_e A^{c\lambda} + 12H^{*\mu}_d H^{*\nu}_e B^{*c\rho\lambda}\Big) \\
&\quad +9\eta^{\mu\sigma\tau}_a \eta^\nu_{b\sigma\tau}H^{*\rho}_d H^{*\lambda}_e \eta^c\Big],
\end{aligned} \tag{A.23}
$$

$$
\begin{aligned}
K^c_{ab,def} &= -2\varepsilon_{\mu\nu\rho\lambda}\Big[\eta^{\sigma\tau\kappa\varsigma}_a \eta_{b\sigma\tau\kappa\varsigma}\left(3H^{*\mu}_d H^{*\nu}_e C^{*\rho\lambda}_f \eta^c + 2H^{*\mu}_d H^{*\nu}_e H^{*\rho}_f A^{c\lambda}\right) \\
&\quad +\eta^{\mu\nu\rho\lambda}_a \eta^{\sigma\tau\kappa}_b H^*_{d\sigma}H^*_{e\tau}H^*_{f\kappa}\eta^c\Big],
\end{aligned} \tag{A.24}
$$

$$
K^c_{ab,defg} = -\varepsilon_{\mu\nu\rho\lambda}\eta^{\sigma\tau\kappa\varsigma}_a \eta_{b\sigma\tau\kappa\varsigma}H^{*\mu}_d H^{*\nu}_e H^{*\rho}_f H^{*\lambda}_g \eta^c. \tag{A.25}
$$

Next, we identify the various notations employed in formula (4.85). The polynomials $X_{A,m_1\ldots m_p}^{abB}$, $X_{A,m_1\ldots m_p}^{abcd}$, $X_{A,m_1\ldots m_p}^{ab}$, $X_{Ac,m_1\ldots m_p}^{ab}$, $X_{Aab,m_1\ldots m_p}$, and $X_{a,m_1\ldots m_p}^{AB}$, with $p=\overline{0,3}$, can be written as

$$\begin{aligned} X_A^{abB} = & \left(C_A^{*}\eta^a - 2C_A^{*\mu}A_\mu^a\right)\eta^b C^B + C_A^{*\mu}\eta^a\eta^b C_\mu^B \\ & +\tfrac{2}{3}\left(V_A^\mu \eta^{*a\nu\rho\lambda} - 3V_A^{*\mu\nu}B^{*a\rho\lambda}\right)\eta^b C^B\varepsilon_{\mu\nu\rho\lambda} \\ & -2\left(V_A^\mu B^{*a\nu\rho} - V_A^{*\mu\nu}A^{a\rho}\right)A^{b\lambda}C^B\varepsilon_{\mu\nu\rho\lambda} + 2V_A^{*\mu\nu}A^{a\rho}\eta^b C^{B\lambda}\varepsilon_{\mu\nu\rho\lambda} \\ & +\left(V_A^{*\mu\nu}V_{\mu\nu}^B + V_A^{*\mu}V_\mu^B\right)\eta^a\eta^b - 2V_A^\mu B^{*a\nu\rho}\eta^b C^{B\lambda}\varepsilon_{\mu\nu\rho\lambda} \\ & +V_A^\mu A^{a\nu}\left(A^{b\rho}C^{B\lambda}\varepsilon_{\mu\nu\rho\lambda} - 2\eta^b V_{\mu\nu}^B\right), \end{aligned} \tag{A.26}$$

$$\begin{aligned} X_{A,m_1}^{abB} = & -\tfrac{1}{2}\left(2H_{m_1}^{*\mu}C_{A\mu}^{*} + C_{m_1}^{*\mu\nu}V_A^{*\rho\lambda}\varepsilon_{\mu\nu\rho\lambda} + \tfrac{1}{3}C_{m_1}^{*\mu\nu\rho}V_A^\lambda\varepsilon_{\mu\nu\rho\lambda}\right)\eta^a\eta^b C^B \\ & +\left[\left(C_{m_1}^{*\mu\nu}V_A^\rho + H_{m_1}^{*\mu}V_A^{*\nu\rho}\right)A^{a\lambda} - 2H_{m_1}^{*\mu}V_A^\nu B^{*a\rho\lambda}\right]\eta^b C^B\varepsilon_{\mu\nu\rho\lambda} \\ & -\tfrac{1}{2}\left(C_{m_1}^{*\mu\nu}V_A^\rho + 2H_{m_1}^{*\mu}V_A^{*\nu\rho}\right)\eta^a\eta^b C^{B\lambda}\varepsilon_{\mu\nu\rho\lambda} + H_{m_1}^{*\mu}V_A^\nu\eta^a\eta^b V_{\mu\nu}^B \\ & +H_{m_1}^{*\mu}V_A^\nu A^{a\rho}\left(A^{b\lambda}C^B + 2\eta^b C^{B\lambda}\right)\varepsilon_{\mu\nu\rho\lambda}, \end{aligned} \tag{A.27}$$

$$\begin{aligned} X_{A,m_1m_2}^{abB} = & -\tfrac{1}{6}\left(3H_{m_1}^{*\mu}H_{m_2}^{*\nu}V_A^{*\rho\lambda} + C_{m_1}^{*[\mu\nu}H_{m_2}^{*\rho]}V_A^\lambda\right)\eta^a\eta^b C^B\varepsilon_{\mu\nu\rho\lambda} \\ & +\tfrac{1}{2}H_{m_1}^{*\mu}H_{m_2}^{*\nu}V_A^\rho\left(2A^{a\lambda}\eta^b C^B - \eta^a\eta^b C^{B\lambda}\right)\varepsilon_{\mu\nu\rho\lambda}, \end{aligned} \tag{A.28}$$

$$X_{A,m_1m_2m_3}^{abB} = -\tfrac{1}{6}H_{m_1}^{*\mu}H_{m_2}^{*\nu}H_{m_3}^{*\rho}V_A^\lambda\eta^a\eta^b C^B\varepsilon_{\mu\nu\rho\lambda}, \tag{A.29}$$

$$\begin{aligned} X_A^{abcd} = & \tfrac{1}{12}C_A^{*}\eta^a\eta^b\eta^c\eta^d - \tfrac{1}{3}V_A^\mu A^{a\nu}A^{b\rho}A^{c\lambda}\eta^d\varepsilon_{\mu\nu\rho\lambda} \\ & -\tfrac{1}{3}\left[C_A^{*\mu}A_\mu^a + \left(V_A^{*\mu\nu}B^{*a\rho\lambda} - \tfrac{1}{3}V_A^\mu\eta^{*a\nu\rho\lambda}\right)\varepsilon_{\mu\nu\rho\lambda}\right]\eta^b\eta^c\eta^d \\ & +\tfrac{1}{2}\left(V_A^{*\mu\nu}A^{a\rho} - 2V_A^\mu B^{*a\nu\rho}\right)A^{b\lambda}\eta^c\eta^d\varepsilon_{\mu\nu\rho\lambda}, \end{aligned} \tag{A.30}$$

$$\begin{aligned} X_{A,m_1}^{abcd} = & -\tfrac{1}{4!}\left[2H_{m_1}^{*\mu}C_{A\mu}^{*} + \left(C_{m_1}^{*\mu\nu}V_A^{*\rho\lambda} + \tfrac{1}{3}C_{m_1}^{*\mu\nu\rho}V_A^\lambda\right)\varepsilon_{\mu\nu\rho\lambda}\right]\eta^a\eta^b\eta^c\eta^d \\ & +\tfrac{1}{6}\left[\left(C_{m_1}^{*\mu\nu}V_A^\rho + 2H_{m_1}^{*\mu}V_A^{*\nu\rho}\right)A^{a\lambda} - 2H_{m_1}^{*\mu}V_A^\nu B^{*a\rho\lambda}\right]\eta^b\eta^c\eta^d\varepsilon_{\mu\nu\rho\lambda} \\ & +\tfrac{1}{2}H_{m_1}^{*\mu}V_A^\nu A^{a\rho}A^{b\lambda}\eta^c\eta^d\varepsilon_{\mu\nu\rho\lambda}, \end{aligned} \tag{A.31}$$

$$X_{A,m_1m_2}^{abcd} = -\tfrac{1}{4!}\left(H_{m_1}^{*\mu}H_{m_2}^{*\nu}V_A^{*\rho\lambda} + \tfrac{1}{3}C_{m_1}^{*[\mu\nu}H_{m_2}^{*\rho]}V_A^\lambda\right)\eta^a\eta^b\eta^c\eta^d\varepsilon_{\mu\nu\rho\lambda}$$

$$+\tfrac{1}{6}H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{\rho}_{A}A^{a\lambda}\eta^{b}\eta^{c}\eta^{d}\varepsilon_{\mu\nu\rho\lambda}, \tag{A.32}$$

$$X^{abcd}_{A,m_1m_2m_3} = -\tfrac{1}{3\cdot 4!}H^{*\mu}_{m_1}H^{*\nu}_{m_2}H^{*\rho}_{m_3}V^{\lambda}_{A}\eta^{a}\eta^{b}\eta^{c}\eta^{d}\varepsilon_{\mu\nu\rho\lambda}, \tag{A.33}$$

$$\begin{aligned} X^{ab}_{A} = {} & \tfrac{1}{12}\left(C^{*}_{A}\eta^{a} - C^{*}_{A\mu}A^{a\mu}\right)C^{b}_{\alpha\beta\gamma\delta}\varepsilon^{\alpha\beta\gamma\delta} - \tfrac{1}{12}C^{*}_{A\mu}\eta^{a}C^{b}_{\nu\rho\lambda}\varepsilon^{\mu\nu\rho\lambda} \\ & +\tfrac{1}{6}V^{*\mu\nu}_{A}\left(12B^{*a\rho\lambda}C^{b}_{\mu\nu\rho\lambda} + \eta^{a}C^{b}_{\mu\nu} - 3A^{a\rho}C^{b}_{\mu\nu\rho}\right) - \tfrac{1}{12}V^{\mu}_{A}\left(2A^{a\nu}C^{b}_{\mu\nu}\right. \\ & \left.+8\eta^{*a\nu\rho\lambda}C^{b}_{\mu\nu\rho\lambda} - 6B^{*a\nu\rho}C^{b}_{\mu\nu\rho} - \eta^{a}H^{b}_{\mu}\right), \end{aligned} \tag{A.34}$$

$$\begin{aligned} X^{ab}_{A,m_1} = {} & \left(-\tfrac{1}{12}H^{*\alpha}_{m_1}C^{*}_{A\alpha}\varepsilon^{\mu\nu\rho\lambda} + C^{*\mu\nu}_{m_1}V^{*\rho\lambda}_{A} + \tfrac{1}{3}C^{*\mu\nu\rho}_{m_1}V^{\lambda}_{A}\right)\eta^{a}C^{b}_{\mu\nu\rho\lambda} \\ & +\tfrac{1}{4}\left(2H^{*\mu}_{m_1}V^{*\nu\rho}_{A} + C^{*\mu\nu}_{m_1}V^{\rho}_{A}\right)\eta^{a}C^{b}_{\mu\nu\rho} \\ & -\tfrac{1}{2}\left(2H^{*\mu}_{m_1}V^{*\nu\rho}_{A} + C^{*\mu\nu}_{m_1}V^{\rho}_{A}\right)A^{a\lambda}C^{b}_{\mu\nu\rho\lambda} \\ & +\tfrac{1}{2}H^{*\mu}_{m_1}V^{\nu}_{A}\left(4B^{*a\rho\lambda}C^{b}_{\mu\nu\rho\lambda} + \tfrac{1}{3}\eta^{a}C^{b}_{\mu\nu} - A^{a\rho}C^{b}_{\mu\nu\rho}\right), \end{aligned} \tag{A.35}$$

$$\begin{aligned} X^{ab}_{A,m_1m_2} = {} & \tfrac{1}{3}\left(3H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{*\rho\lambda}_{A} + C^{*[\mu\nu}_{m_1}H^{\rho]}_{m_2}V^{\lambda}_{A}\right)\eta^{a}C^{b}_{\mu\nu\rho\lambda} \\ & +\tfrac{1}{4}H^{*\mu}_{m_1}H^{*\nu}_{m_2}\left(V^{\rho}_{A}\eta^{a}C^{b}_{\mu\nu\rho} - 4V^{\rho}_{A}A^{a\lambda}C^{b}_{\mu\nu\rho\lambda}\right), \end{aligned} \tag{A.36}$$

$$X^{ab}_{A,m_1m_2m_3} = \tfrac{1}{3}H^{*\mu}_{m_1}H^{*\nu}_{m_2}H^{\rho}_{m_3}V^{\lambda}_{A}\eta^{a}C^{b}_{\mu\nu\rho\lambda}, \tag{A.37}$$

$$\begin{aligned} X^{ab}_{Ac} = {} & \tfrac{1}{4!}\left(C^{*}_{A}\eta^{a} - 2C^{*}_{A\alpha}A^{a\alpha}\right)\eta^{b}\eta_{c\mu\nu\rho\lambda}\varepsilon^{\mu\nu\rho\lambda} - \tfrac{1}{4!}C^{*}_{A\mu}\eta^{a}\eta^{b}\eta_{c\nu\rho\lambda}\varepsilon^{\mu\nu\rho\lambda} \\ & +V^{*\mu\nu}_{A}\left(2B^{*a\rho\lambda}\eta^{b} - A^{a\rho}A^{b\lambda}\right)\eta_{c\mu\nu\rho\lambda} \\ & +\tfrac{1}{12}\left(V^{*\mu\nu}_{A}\eta^{a} - 2V^{\mu}_{A}A^{a\nu}\right)\eta^{b}B_{c\mu\nu} \\ & -\tfrac{1}{2}\left(V^{*\mu\nu}_{A}A^{a\rho} - V^{\mu}_{A}B^{*a\nu\rho}\right)\eta^{b}\eta_{c\mu\nu\rho} \\ & -2V^{\mu}_{A}\left(\tfrac{1}{3}\eta^{*a\nu\rho\lambda}\eta^{b} - B^{*a\nu\rho}A^{b\lambda}\right)\eta_{c\mu\nu\rho\lambda} \\ & -\tfrac{1}{12}V^{\mu}_{A}\eta^{a}\eta^{b}A^{*}_{c\mu} - \tfrac{1}{4}V^{\mu}_{A}A^{a\nu}A^{b\rho}\eta_{c\mu\nu\rho}, \end{aligned} \tag{A.38}$$

$$\begin{aligned} X^{ab}_{Ac,m_1} = {} & -\tfrac{1}{4!}\left(H^{*\alpha}_{m_1}C^{*}_{A\alpha}\varepsilon^{\mu\nu\rho\lambda} - 12C^{*\mu\nu}_{m_1}V^{*\rho\lambda}_{A} - 4C^{*\mu\nu\rho}_{m_1}V^{\lambda}_{A}\right)\eta^{a}\eta^{b}\eta_{c\mu\nu\rho\lambda} \\ & +\tfrac{1}{4}\left(2H^{*\mu}_{m_1}V^{*\nu\rho}_{A} + C^{*\mu\nu}_{m_1}V^{\rho}_{A}\right)\eta^{a}\eta^{b}\eta_{c\mu\nu\rho} + \tfrac{1}{12}H^{*\mu}_{m_1}V^{\nu}_{A}\eta^{a}\eta^{b}B_{c\mu\nu} \\ & -\left[\left(2H^{*\mu}_{m_1}V^{*\nu\rho}_{A} + C^{*\mu\nu}_{m_1}V^{\rho}_{A}\right)A^{a\lambda} - 2H^{*\mu}_{m_1}V^{\nu}_{A}B^{*a\rho\lambda}\right]\eta^{b}\eta_{c\mu\nu\rho\lambda} \\ & -\tfrac{1}{2}H^{*\mu}_{m_1}V^{\nu}_{A}A^{a\rho}\left(\eta^{b}\eta_{c\mu\nu\rho} + 2A^{b\lambda}\eta_{c\mu\nu\rho\lambda}\right), \end{aligned} \tag{A.39}$$

$$X^{ab}_{Ac,m_1m_2} = \tfrac{1}{6}\left(3H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{*\rho\lambda}_{A} + C^{*[\mu\nu}_{m_1}H^{*\rho]}_{m_2}V^{\lambda}_{A}\right)\eta^a\eta^b\eta_{c\mu\nu\rho\lambda} + \tfrac{1}{8}H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{\rho}_{A}\left(\eta^a\eta^b\eta_{c\mu\nu\rho} - 8A^{a\lambda}\eta^b\eta_{c\mu\nu\rho\lambda}\right), \tag{A.40}$$

$$X^{ab}_{Ac,m_1m_2m_3} = \tfrac{1}{6}H^{*\mu}_{m_1}H^{*\nu}_{m_2}H^{*\rho}_{m_3}V^{\lambda}_{A}\eta^a\eta^b\eta_{c\mu\nu\rho\lambda}, \tag{A.41}$$

$$X_{Aab} = -\left(C^{*}_{A}\eta_{a\mu\nu\rho\lambda} + 2C^{*}_{A\mu}\eta_{a\nu\rho\lambda}\right)\eta^{\mu\nu\rho\lambda}_{b} + \tfrac{3}{4}V^{*\mu\nu}_{A}\eta^{\ \ \rho}_{a\alpha\beta}\eta^{\lambda\alpha\beta}_{b}\varepsilon_{\mu\nu\rho\lambda} + 2\left(V^{*\alpha\beta}_{A}B_{a\alpha\beta} - \tfrac{1}{12}V^{\alpha}_{A}A^{*}_{a\alpha}\right)\eta_{b\mu\nu\rho\lambda}\varepsilon^{\mu\nu\rho\lambda} - \tfrac{1}{12}V^{\alpha}_{A}B_{a\alpha\mu}\eta_{b\nu\rho\lambda}\varepsilon^{\mu\nu\rho\lambda}, \tag{A.42}$$

$$X_{Aab,m_1} = \tfrac{1}{6}\left(6H^{*\mu}_{m_1}C^{*}_{A\mu} + 3C^{*\mu\nu}_{m_1}V^{*\rho\lambda}_{A}\varepsilon_{\mu\nu\rho\lambda} + C^{*\mu\nu\rho}_{m_1}V^{\lambda}_{A}\varepsilon_{\mu\nu\rho\lambda}\right)\eta_{a\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta}_{b} + \tfrac{1}{4}\left(2H^{*\mu}_{m_1}V^{*\nu\rho}_{A} + C^{*\mu\nu}_{m_1}V^{\rho}_{A}\right)\eta_{a\mu\nu\rho}\eta_{b\alpha\beta\gamma\delta}\varepsilon^{\alpha\beta\gamma\delta} + \tfrac{3}{4}H^{*\mu}_{m_1}V^{\nu}_{A}\eta^{\ \ \rho}_{a\alpha\beta}\eta^{\lambda\alpha\beta}_{b}\varepsilon_{\mu\nu\rho\lambda} + \tfrac{1}{6}H^{*\mu}_{m_1}V^{\nu}_{A}B_{a\mu\nu}\eta_{b\alpha\beta\gamma\delta}\varepsilon^{\alpha\beta\gamma\delta}, \tag{A.43}$$

$$X_{Aab,m_1m_2} = \tfrac{1}{6}\left(3H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{*\rho\lambda}_{A} + C^{*[\mu\nu}_{m_1}H^{*\rho]}_{m_2}V^{\lambda}_{A}\right)\eta_{a\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta}_{b}\varepsilon_{\mu\nu\rho\lambda} + \tfrac{1}{4}H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{\rho}_{A}\eta_{a\mu\nu\rho}\eta_{b\alpha\beta\gamma\delta}\varepsilon^{\alpha\beta\gamma\delta}, \tag{A.44}$$

$$X_{Aab,m_1m_2m_3} = \tfrac{1}{6}H^{*\mu}_{m_1}H^{*\nu}_{m_2}H^{*\rho}_{m_3}V^{\lambda}_{A}\eta_{a\alpha\beta\gamma\delta}\eta^{\alpha\beta\gamma\delta}_{b}\varepsilon_{\mu\nu\rho\lambda}, \tag{A.45}$$

$$X^{AB}_{a} = -4\left(C^{*A}C^{B} + C^{*A}_{\alpha}C^{B\alpha}\right)\eta_{a\mu\nu\rho\lambda}\varepsilon^{\mu\nu\rho\lambda} + 4\left(C^{*A}_{\mu}C^{B}\varepsilon^{\mu\nu\rho\lambda} - 6V^{*A\nu\rho}C^{B\lambda}\right)\eta_{a\nu\rho\lambda} - 8V^{*A\mu\nu}C^{B}B_{a\mu\nu} + 8V^{A}_{\mu}C^{B}A^{*\mu}_{a} - \left(V^{*A}_{\alpha}V^{B\alpha} + 4V^{*A\alpha\beta}V^{B}_{\alpha\beta}\right)\eta_{a\mu\nu\rho\lambda}\varepsilon^{\mu\nu\rho\lambda} + 4V^{A\alpha}V^{B}_{\alpha\mu}\varepsilon^{\mu\nu\rho\lambda}\eta_{a\nu\rho\lambda} - 8V^{A\mu}C^{B\nu}B_{a\mu\nu}, \tag{A.46}$$

$$X^{AB}_{a,m_1} = 4\left(H^{*\alpha}_{m_1}C^{*A}_{\alpha}\varepsilon^{\mu\nu\rho\lambda} - 12C^{*\mu\nu}_{m_1}V^{*A\rho\lambda} - 4C^{*\mu\nu\rho}_{m_1}V^{A\lambda}\right)C^{B}\eta_{a\mu\nu\rho\lambda} - 12\left(2H^{*\mu}_{m_1}V^{*A\nu\rho} + C^{*\mu\nu}_{m_1}V^{A\rho}\right)\left(C^{B}\eta_{a\mu\nu\rho} + 4C^{B\lambda}\eta_{a\mu\nu\rho\lambda}\right) - 4H^{*\mu}_{m_1}V^{A\nu}\left(6C^{B\rho}\eta_{a\mu\nu\rho} + V^{B}_{\mu\nu}\eta_{a\alpha\beta\gamma\delta}\varepsilon^{\alpha\beta\gamma\delta} + 2C^{B}B_{a\mu\nu}\right), \tag{A.47}$$

$$X^{AB}_{a,m_1m_2} = -16\left(3H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{*A\rho\lambda} + C^{*[\mu\nu}_{m_1}H^{*\rho]}_{m_2}V^{A\lambda}\right)C^{B}\eta_{a\mu\nu\rho\lambda} - 12H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{A\rho}\left(C^{B}\eta_{a\mu\nu\rho} + 4C^{B\lambda}\eta_{a\mu\nu\rho\lambda}\right), \tag{A.48}$$

$$X^{AB}_{a,m_1m_2m_3} = -16H^{*\mu}_{m_1}H^{*\nu}_{m_2}H^{*\rho}_{m_3}V^{\lambda}_{A}C^{B}\eta_{a\mu\nu\rho\lambda}. \tag{A.49}$$

The objects denoted by $X^{aABC}_{m_1\ldots m_p}$, $X^{abc}_{AB,m_1\ldots m_p}$, $X^{a}_{AB,m_1\ldots m_p}$, and $X^{b}_{ABa,m_1\ldots m_p}$, with $p=\overline{0,2}$, read as

$$\begin{aligned} X^{aABC} = & -\tfrac{1}{4!}\left(2C^{*A\mu}V^{B}_{\mu}+V^{*A\mu\nu}V^{*B\rho\lambda}\varepsilon_{\mu\nu\rho\lambda}\right)C^{C}\eta^{a} \\ & +\tfrac{1}{12}V^{*A\mu\nu}V^{B\rho}\left(C^{C}A^{a\lambda}-C^{C\lambda}\eta^{a}\right)\varepsilon_{\mu\nu\rho\lambda} \\ & +\tfrac{1}{4!}V^{A\mu}V^{B\nu}C^{C}B^{*a\rho\lambda}\varepsilon_{\mu\nu\rho\lambda} \\ & -\tfrac{1}{4!}V^{A\mu}V^{B\nu}\left(C^{C\rho}A^{a\lambda}\varepsilon_{\mu\nu\rho\lambda}+V^{C}_{\mu\nu}\eta^{a}\right), \end{aligned} \tag{A.50}$$

$$\begin{aligned} X^{aABC}_{m_1} = & \tfrac{1}{2\cdot 4!}\left(C^{*\mu\nu}_{m_1}V^{A\rho}+4H^{*\mu}_{m_1}V^{*A\nu\rho}\right)V^{B\lambda}C^{C}\eta^{a}\varepsilon_{\mu\nu\rho\lambda} \\ & +\tfrac{1}{4!}H^{*\mu}_{m_1}V^{A\nu}V^{B\rho}\left(C^{C}A^{a\lambda}-C^{C\lambda}\eta^{a}\right)\varepsilon_{\mu\nu\rho\lambda}, \end{aligned} \tag{A.51}$$

$$X^{aABC}_{m_1m_2} = \tfrac{1}{2\cdot 4!}H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{A\rho}V^{B\lambda}C^{C}\eta^{a}\varepsilon_{\mu\nu\rho\lambda}, \tag{A.52}$$

$$\begin{aligned} X^{abc}_{AB} = & \tfrac{1}{2\cdot 4!}\left[\left(V^{*\mu\nu}_{A}V^{\rho}_{B}A^{a\lambda}+V^{\mu}_{A}V^{\nu}_{B}B^{*a\rho\lambda}\right)\eta^{b}-V^{\mu}_{A}V^{\nu}_{B}A^{a\rho}A^{b\lambda}\right]\eta^{c}\varepsilon_{\mu\nu\rho\lambda} \\ & -\tfrac{1}{6\cdot 4!}\left(2C^{*}_{A\mu}V^{\mu}_{B}+V^{*\mu\nu}_{A}V^{*\rho\lambda}_{B}\varepsilon_{\mu\nu\rho\lambda}\right)\eta^{a}\eta^{b}\eta^{c}, \end{aligned} \tag{A.53}$$

$$\begin{aligned} X^{abc}_{AB,m_1} = & \tfrac{1}{12\cdot 4!}\left[\left(4H^{*\mu}_{m_1}V^{*\nu\rho}_{A}V^{\lambda}_{B}+C^{*\mu\nu}_{m_1}V^{\rho}_{A}V^{\lambda}_{B}\right)\eta^{a}\right. \\ & \left.+6H^{*\mu}_{m_1}V^{\nu}_{A}V^{\rho}_{B}A^{a\lambda}\right]\eta^{b}\eta^{c}\varepsilon_{\mu\nu\rho\lambda}, \end{aligned} \tag{A.54}$$

$$X^{abc}_{AB,m_1m_2} = \tfrac{1}{12\cdot 4!}H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{\rho}_{A}V^{\lambda}_{B}\eta^{a}\eta^{b}\eta^{c}\varepsilon_{\mu\nu\rho\lambda}, \tag{A.55}$$

$$\begin{aligned} X^{a}_{AB} = & -\tfrac{1}{12\cdot 4!}\left(C^{*}_{A\alpha}V^{\alpha}_{B}\varepsilon^{\mu\nu\rho\lambda}-12V^{*\mu\nu}_{A}V^{*\rho\lambda}_{B}\right)C^{a}_{\mu\nu\rho\lambda} \\ & -\tfrac{1}{2\cdot 4!}V^{*\mu\nu}_{A}V^{\rho}_{B}C^{a}_{\mu\nu\rho}-\tfrac{1}{2\cdot 4!}V^{\mu}_{A}V^{\nu}_{B}C^{a}_{\mu\nu}, \end{aligned} \tag{A.56}$$

$$\begin{aligned} X^{a}_{AB,m_1} = & -\tfrac{1}{12}\left(H^{*\mu}_{m_1}V^{*\nu\rho}_{A}V^{\lambda}_{B}+\tfrac{1}{4}C^{*\mu\nu}_{m_1}V^{\rho}_{A}V^{\lambda}_{B}\right)C^{a}_{\mu\nu\rho\lambda} \\ & -\tfrac{1}{4\cdot 4!}H^{*\mu}_{m_1}V^{\nu}_{A}V^{\rho}_{B}C^{a}_{\mu\nu\rho}, \end{aligned} \tag{A.57}$$

$$X^{a}_{AB,m_1m_2} = -\tfrac{1}{2\cdot 4!}H^{*\mu}_{m_1}H^{*\nu}_{m_2}V^{\rho}_{A}V^{\lambda}_{B}C^{a}_{\mu\nu\rho\lambda}, \tag{A.58}$$

$$X^{b}_{ABa} = -\tfrac{1}{12\cdot 4!}\left(C^{*}_{A\alpha}V^{\alpha}_{B}\varepsilon^{\mu\nu\rho\lambda}-12V^{*\mu\nu}_{A}V^{*\rho\lambda}_{B}\right)\eta_{a\mu\nu\rho\lambda}\eta^{b}$$

$$
\begin{aligned}
&+\tfrac{1}{2\cdot 4!}V_A^{*\mu\nu}V_B^{\rho}\left(\eta_{a\mu\nu\rho}\eta^b-4\eta_{a\mu\nu\rho\lambda}A^{b\lambda}\right)-\tfrac{1}{4!}V_A^{\mu}V_B^{\nu}\eta_{a\mu\nu\rho\lambda}B^{*b\rho\lambda}\\
&-\tfrac{1}{12\cdot 4!}V_A^{\mu}V_B^{\nu}\left(B_{a\mu\nu}\eta^b+3\eta_{a\mu\nu\rho}A^{b\rho}\right),
\end{aligned}
\tag{A.59}
$$

$$
\begin{aligned}
X^b_{ABa,m_1} \;=\;& -\tfrac{1}{12}\left(H^{*\mu}_{m_1}V_A^{*\nu\rho}+\tfrac{1}{4}C^{*\mu\nu}_{m_1}V_A^{\rho}\right)V_B^{\lambda}\eta_{a\mu\nu\rho\lambda}\eta^b\\
&+\tfrac{1}{4\cdot 4!}H^{*\mu}_{m_1}V_A^{\nu}V_B^{\rho}\left(\eta_{a\mu\nu\rho}\eta^b-4\eta_{a\mu\nu\rho\lambda}A^{b\lambda}\right),
\end{aligned}
\tag{A.60}
$$

$$
X^b_{ABa,m_1m_2}=-\tfrac{1}{2\cdot 4!}H^{*\mu}_{m_1}H^{*\nu}_{m_2}V_A^{\rho}V_B^{\lambda}\eta_{a\mu\nu\rho\lambda}\eta^b. \tag{A.61}
$$

In the end of this section we list the remaining type-X objects from (4.85), namely $X_{ABCD,m_1\ldots m_p}$, $X^{ab}_{ABC,m_1\ldots m_p}$, and $X^{ABC}_{a,m_1\ldots m_p}$, with $p=\overline{0,1}$, as well as X^a_{ABCD}:

$$
X_{ABCD}=\tfrac{1}{12}\left(V_A^{*\mu\nu}V_B^{\rho}V_C^{\lambda}C_D+\tfrac{1}{3}V_A^{\mu}V_B^{\nu}V_C^{\rho}C_D^{\lambda}\right)\varepsilon_{\mu\nu\rho\lambda}, \tag{A.62}
$$

$$
X_{ABCD,m_1}=\tfrac{2}{3\cdot 4!}H^{*\mu}_{m_1}V_A^{\nu}V_B^{\rho}V_C^{\lambda}C_D\varepsilon_{\mu\nu\rho\lambda}, \tag{A.63}
$$

$$
X^{ab}_{ABC}=\tfrac{1}{4!}\left(V_A^{*\mu\nu}V_B^{\rho}V_C^{\lambda}\eta^a\eta^b-\tfrac{2}{3}V_A^{\mu}V_B^{\nu}V_C^{\rho}A^{a\lambda}\eta^b\right)\varepsilon_{\mu\nu\rho\lambda}, \tag{A.64}
$$

$$
X^{ab}_{ABC,m_1}=\tfrac{1}{3\cdot 4!}H^{*\mu}_{m_1}V_A^{\nu}V_B^{\rho}V_C^{\lambda}\eta^a\eta^b\varepsilon_{\mu\nu\rho\lambda}, \tag{A.65}
$$

$$
X^{ABC}_a=-\tfrac{1}{12}V^{*A\mu\nu}V^{B\rho}V^{C\lambda}\eta_{a\mu\nu\rho\lambda}-\tfrac{1}{6\cdot 4!}V^{A\mu}V^{B\nu}V^{C\rho}\eta_{a\mu\nu\rho}, \tag{A.66}
$$

$$
X^{ABC}_{a,m_1}=-\tfrac{2}{3\cdot 4!}H^{*\mu}_{m_1}V^{A\nu}V^{B\rho}V^{C\lambda}\eta_{a\mu\nu\rho\lambda}, \tag{A.67}
$$

$$
X^a_{ABCD}=-\tfrac{1}{3\cdot 4!}V_A^{\mu}V_B^{\nu}V_C^{\rho}V_D^{\lambda}\eta^a\varepsilon_{\mu\nu\rho\lambda}. \tag{A.68}
$$

B Gauge generators of the deformed model

From the terms of antighost number 1 present in (4.111) we determine the deformed gauge generators that produce the deformed gauge transformations (5.7)–(5.12). We added a supplementary index between parentheses to the gauge generators such as to distinguish among the fields to which the gauge generators are associated with. We list below only the nonvanishing generators of the various fields, which read as:

$$
(\bar{Z}_{a(\varphi)})_b=-\lambda W_{ab}, \tag{B.1}
$$

$$
(\bar{Z}^a_{\mu(H)})_b \;=\; \tfrac{\lambda}{2}\varepsilon_{\mu\nu\rho\lambda}\left[\left(-\tfrac{1}{12}\frac{\partial M_{bcde}}{\partial\varphi_a}A^{c\nu}+\frac{\partial f^A_{bde}}{\partial\varphi_a}V_A^{\nu}\right)A^{d\rho}\right.
$$

$$
\begin{aligned}
&+\frac{\partial g^{AB}_{be}}{\partial\varphi_a}V^\nu_A V^\rho_B\Big] A^{e\lambda}+\lambda\left[-\frac{\partial W_{bc}}{\partial\varphi_a}H^c_\mu+\frac{\partial f^A_{bB}}{\partial\varphi_a}V^\nu_A V^B_{\mu\nu}\right.\\
&\left.+\left(\frac{\partial M^d_{bc}}{\partial\varphi_a}A^{c\nu}+\tfrac{1}{12}\frac{\partial f^{Ad}{}_b}{\partial\varphi_a}V^\nu_A\right)B_{d\mu\nu}\right],
\end{aligned}
\tag{B.2}
$$

$$
(\bar Z^a_{\mu(H)})^{\alpha\beta}_b=-\delta^a_b\partial^{[\alpha}\delta^{\beta]}_\mu+\lambda\left(\frac{\partial W_{bc}}{\partial\varphi_a}A^{c[\alpha}\delta^{\beta]}_\mu-\tfrac{1}{12}\frac{\partial f_{Ab}}{\partial\varphi_a}V^{A[\alpha}\delta^{\beta]}_\mu\right),
\tag{B.3}
$$

$$
\begin{aligned}
(\bar Z^a_{\mu(H)})^b_{\alpha\beta\gamma} &= -\tfrac{\lambda}{2}\frac{\partial M^b_{cd}}{\partial\varphi_a}\sigma_{\mu[\alpha}A^c_\beta A^d_{\gamma]}+2\lambda\frac{\partial M^{bc}}{\partial\varphi_a}\sigma_{\mu\rho}B^{\rho\lambda}_c\varepsilon_{\lambda\alpha\beta\gamma}\\
&\quad+\tfrac{\lambda}{4!}\left(\frac{\partial f^b_{Ac}}{\partial\varphi_a}\sigma_{\mu[\alpha}V^A_\beta A^c_{\gamma]}-\frac{\partial g^b_{AB}}{\partial\varphi_a}\sigma_{\mu[\alpha}V^A_\beta V^B_{\gamma]}\right),
\end{aligned}
\tag{B.4}
$$

$$
(\bar Z^a_{\mu(H)})^\sigma_A=\lambda\varepsilon_{\mu\nu\rho\lambda}\sigma^{\lambda\sigma}\left(\frac{\partial f_{bAB}}{\partial\varphi_a}V^{B\nu}A^{b\rho}+\tfrac12\frac{\partial g^{BC}{}_A}{\partial\varphi_a}V^\nu_B V^\rho_C\right),
\tag{B.5}
$$

$$
(\bar Z^a_{\mu(A)})_b=\delta^a_b\partial_\mu-\lambda M^a_{bc}A^c_\mu-\tfrac{\lambda}{12}f^a_{Ab}V^A_\mu,
\tag{B.6}
$$

$$
(\bar Z^a_{\mu(A)})^b_{\alpha\beta\gamma}=-2\lambda M^{ab}\varepsilon_{\mu\alpha\beta\gamma},
\tag{B.7}
$$

$$
\begin{aligned}
(\bar Z^{\mu\nu}_{a(B)})_b &= \lambda\varepsilon^{\mu\nu\rho\lambda}\left(\tfrac18 M_{abcd}A^c_\rho A^d_\lambda+f_{Aabc}V^A_\rho A^c_\lambda-\tfrac12 g_{ABab}V^A_\rho V^B_\lambda\right)\\
&\quad-\lambda M^c_{ab}B^{\mu\nu}_c,
\end{aligned}
\tag{B.8}
$$

$$
(\bar Z^{\mu\nu}_{a(B)})^{\alpha\beta}_b=\lambda W_{ab}\sigma^{\mu[\alpha}\sigma^{\beta]\nu},
\tag{B.9}
$$

$$
(\bar Z^{\mu\nu}_{a(B)})^b_{\alpha\beta\gamma}=-\tfrac12\delta^b_a\partial_{[\alpha}\delta^\mu_\beta\delta^\nu_{\gamma]}-\tfrac{\lambda}{2}\left(M^b_{ac}\delta^\mu_{[\alpha}\delta^\nu_\beta A^c_{\gamma]}+\tfrac{1}{12}f^b_{Aa}\delta^\mu_{[\alpha}\delta^\nu_\beta V^A_{\gamma]}\right),
\tag{B.10}
$$

$$
(\bar Z^{\mu\nu}_{a(B)})^\lambda_A=-\lambda\varepsilon^{\mu\nu\rho\lambda}f_{aAB}V^B_\rho,
\tag{B.11}
$$

$$
(\bar Z^A_{\mu(V)})_a=\lambda f^A_{aB}V^B_\mu,
\tag{B.12}
$$

$$
(\bar Z^A_{\mu\nu(V)})_a=\lambda f^A_{aB}V^B_{\mu\nu}+\tfrac{\lambda}{12}f^{Ab}{}_a B_{b\mu\nu}+\lambda\varepsilon_{\mu\nu\rho\lambda}\left(\tfrac12 f^A_{abc}A^{b\rho}+g^{AB}_{ac}V^\rho_B\right)A^{c\lambda},
\tag{B.13}
$$

$$
(\bar Z^A_{\mu\nu(V)})^{\alpha\beta}_a=\tfrac{\lambda}{4!}f^A_a\delta^{[\alpha}_\mu\delta^{\beta]}_\nu,
\tag{B.14}
$$

$$
(\bar Z^A_{\mu\nu(V)})^a_{\alpha\beta\gamma}=\tfrac{\lambda}{4!}\left(f^{Aa}{}_b A^b_\sigma-g^{aAB}V_{B\sigma}\right)\sigma_{\mu\rho}\sigma_{\nu\lambda}\delta^\rho_{[\alpha}\delta^\lambda_\beta\delta^\sigma_{\gamma]},
\tag{B.15}
$$

$$
(\bar Z^A_{\mu\nu(V)})_{B\lambda}=\varepsilon_{\mu\nu\rho\lambda}\left(\delta^A_B\partial^\rho-\lambda f^A_{aB}A^{a\rho}+\lambda g^{AC}{}_B V^\rho_C\right).
\tag{B.16}
$$

C Reducibility of the deformed gauge transformations

From the terms of antighost number 2 in (4.111) that are simultaneously linear in the ghosts for ghosts and in the antifields of the ghosts we identify the first-order reducibility functions for the coupled model as

$$(\bar{Z}^{(1)a}_{\alpha\beta})^{\mu\nu\rho}_{b} = -\tfrac{1}{2}\left(\delta^{a}_{b}\partial^{[\mu}\delta^{\nu}_{\alpha}\delta^{\rho]}_{\beta} - \lambda\frac{\partial W_{bc}}{\partial\varphi_a}A^{c[\mu}\delta^{\nu}_{\alpha}\delta^{\rho]}_{\beta}\right) - \tfrac{\lambda}{2\cdot 4!}\frac{\partial f^{A}_{b}}{\partial\varphi_a}\delta^{[\mu}_{\alpha}\delta^{\nu}_{\beta}\delta^{\rho]}_{\gamma}V^{\gamma}_{A}, \tag{C.1}$$

$$\begin{aligned}(\bar{Z}^{(1)a}_{\alpha\beta})^{b}_{\mu\nu\rho\lambda} = {} & \tfrac{\lambda}{8}\frac{\partial M^{b}_{cd}}{\partial\varphi_a}\sigma_{\alpha'[\alpha}\sigma_{\beta]\beta'}\delta^{\alpha'}_{[\mu}\delta^{\beta'}_{\nu}A^{c}_{\rho}A^{d}_{\lambda]} + \lambda\varepsilon_{\mu\nu\rho\lambda}\frac{\partial M^{bc}}{\partial\varphi_a}B_{c\alpha\beta} \\ & -\tfrac{\lambda}{4\cdot 4!}\varepsilon_{\mu\nu\rho\lambda}\varepsilon_{\alpha\beta\gamma\delta}\left(\frac{\partial g^{bAB}}{\partial\varphi_a}V^{\gamma}_{A}V^{\delta}_{B} - 2\frac{\partial f^{Ab}_{\ \ c}}{\partial\varphi_a}V^{\gamma}_{A}A^{c\delta}\right),\end{aligned} \tag{C.2}$$

$$(\bar{Z}^{(1)a}_{\alpha\beta})_{A} = \tfrac{\lambda}{2}\varepsilon_{\alpha\beta\rho\lambda}\left(\frac{\partial f^{B}_{bA}}{\partial\varphi_a}V^{\rho}_{B}A^{b\lambda} - \tfrac{1}{2}\frac{\partial g^{BC}_{\ \ \ A}}{\partial\varphi_a}V^{\rho}_{B}V^{\lambda}_{C}\right), \tag{C.3}$$

$$\begin{aligned}(\bar{Z}^{(1)\alpha\beta\gamma}_{a})^{b}_{\mu\nu\rho\lambda} = {} & -\tfrac{1}{6}\left(\delta^{b}_{a}\partial_{[\mu}\delta^{\alpha}_{\nu}\delta^{\beta}_{\rho}\delta^{\gamma}_{\lambda]} + \lambda M^{b}_{ac}A^{c}_{[\mu}\delta^{\alpha}_{\nu}\delta^{\beta}_{\rho}\delta^{\gamma}_{\lambda]}\right) \\ & +\tfrac{\lambda}{3\cdot 4!}f^{Ab}_{\ \ a}\delta^{\alpha}_{[\mu}\delta^{\beta}_{\nu}\delta^{\gamma}_{\rho}\delta^{\delta}_{\lambda]}V_{A\delta},\end{aligned} \tag{C.4}$$

$$(\bar{Z}^{(1)\alpha\beta\gamma}_{a})^{\mu\nu\rho}_{b} = -\tfrac{\lambda}{3}W_{ab}\left(\sigma^{\alpha[\mu}\sigma^{\nu]\beta}\sigma^{\rho\gamma} + \sigma^{\alpha[\nu}\sigma^{\rho]\beta}\sigma^{\mu\gamma} + \sigma^{\alpha[\rho}\sigma^{\mu]\beta}\sigma^{\nu\gamma}\right), \tag{C.5}$$

$$(\bar{Z}^{(1)\alpha\beta\gamma}_{a})_{A} = -\tfrac{\lambda}{3}\varepsilon^{\alpha\beta\gamma\delta}f^{B}_{aA}V_{B\delta}, \tag{C.6}$$

$$(\bar{Z}^{(1)A}_{\mu})_{B} = \delta^{A}_{B}\partial_{\mu} - \lambda f^{A}_{aB}A^{a}_{\mu} + \lambda g^{AC}_{\ \ \ B}V_{C\mu}, \tag{C.7}$$

$$(\bar{Z}^{(1)A}_{\mu})^{\alpha\beta\gamma}_{a} = \tfrac{\lambda}{4!}f^{A}_{a}\sigma_{\mu\nu}\varepsilon^{\nu\alpha\beta\gamma}, \tag{C.8}$$

$$(\bar{Z}^{(1)A}_{\mu})^{a}_{\alpha\beta\gamma\delta} = -\tfrac{\lambda}{4!}\varepsilon_{\alpha\beta\gamma\delta}\left(f^{Aa}_{\ \ b}A^{b}_{\mu} - g^{aAB}V_{B\mu}\right), \tag{C.9}$$

$$(\bar{Z}^{(1)a})^{b}_{\alpha\beta\gamma\delta} = -2\lambda\varepsilon_{\alpha\beta\gamma\delta}M^{ab}. \tag{C.10}$$

The first-order reducibility relations of the coupled theory result from the components of (4.111) with the antighost number equal to 2 that are simultaneously linear in the ghosts for ghosts and quadratic in the antifields of the original fields, being expressed in De Witt condensed form as

$$(\bar{Z}^{a}_{\mu(A)})_{e}(\bar{Z}^{(1)e})^{b}_{\alpha\beta\gamma\delta} + (\bar{Z}^{a}_{\mu(A)})^{e}_{\nu\rho\lambda}(\bar{Z}^{(1)\nu\rho\lambda}_{e})^{b}_{\alpha\beta\gamma\delta} = -2\lambda\varepsilon_{\alpha\beta\gamma\delta}\frac{\partial M^{ab}}{\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c\mu}}, \tag{C.11}$$

$$
\begin{aligned}
&(\bar{Z}^{\mu\nu}_{a(B)})^{e}_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_{e})^{\alpha\beta\gamma}_{b}+(\bar{Z}^{\mu\nu}_{a(B)})^{\rho\lambda}_{e}(\bar{Z}^{(1)e}_{\rho\lambda})^{\alpha\beta\gamma}_{b}+(\bar{Z}^{\mu\nu}_{a(B)})^{\sigma}_{A}(\bar{Z}^{(1)A}_{\sigma})^{\alpha\beta\gamma}_{b}\\
={}&\lambda\frac{\partial W_{ab}}{\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c}_{\rho}}\sigma^{\mu\mu'}\sigma^{\nu\nu'}\delta^{[\alpha}_{\mu'}\delta^{\beta}_{\nu'}\delta^{\gamma]}_{\rho},
\end{aligned}
\tag{C.12}
$$

$$
\begin{aligned}
&(\bar{Z}^{\mu\nu}_{a(B)})_{e}(\bar{Z}^{(1)e})^{b}_{\alpha\beta\gamma\delta}+(\bar{Z}^{\mu\nu}_{a(B)})^{e}_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_{e})^{b}_{\alpha\beta\gamma\delta}\\
&+(\bar{Z}^{\mu\nu}_{a(B)})^{\rho\lambda}_{e}(\bar{Z}^{(1)e}_{\rho\lambda})^{b}_{\alpha\beta\gamma\delta}+(\bar{Z}^{\mu\nu}_{a(B)})^{\sigma}_{A}(\bar{Z}^{(1)A}_{\sigma})^{b}_{\alpha\beta\gamma\delta}\\
={}&-\tfrac{\lambda}{2}\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta}\delta^{\rho}_{\gamma}\delta^{\lambda}_{\delta]}\left(\frac{\partial M^{b}_{ac}}{\partial\varphi_d}\frac{\delta S^{\mathrm{L}}}{\delta H^{d\rho}}A^{c}_{\lambda}+M^{b}_{ac}\frac{\delta S^{\mathrm{L}}}{\delta B^{\rho\lambda}_{c}}\right)\\
&-\tfrac{\lambda}{4!}\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta}\delta^{\rho}_{\gamma}\delta^{\lambda}_{\delta]}\left(f^{Ab}_{\ \ a}\frac{\delta S^{\mathrm{L}}}{\delta V^{A\rho\lambda}}+\frac{\partial f^{Ab}_{\ \ a}}{\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c\rho}}V_{A\lambda}\right),
\end{aligned}
\tag{C.13}
$$

$$
\begin{aligned}
&(\bar{Z}^{\mu\nu}_{a(B)})^{e}_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_{e})_{A}+(\bar{Z}^{\mu\nu}_{a(B)})^{\rho\lambda}_{e}(\bar{Z}^{(1)e}_{\rho\lambda})_{A}+(\bar{Z}^{\mu\nu}_{a(B)})^{\sigma}_{B}(\bar{Z}^{(1)B}_{\sigma})_{A}\\
={}&\lambda\varepsilon^{\mu\nu\rho\lambda}\left(f^{B}_{aA}\frac{\delta S^{\mathrm{L}}}{\delta V^{B\rho\lambda}}+\frac{\partial f^{B}_{aA}}{\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c\rho}}V_{B\lambda}\right),
\end{aligned}
\tag{C.14}
$$

$$
\begin{aligned}
&(\bar{Z}^{A}_{\mu\nu(V)})^{\sigma}_{C}(\bar{Z}^{(1)C}_{\sigma})_{B}+(\bar{Z}^{A}_{\mu\nu(V)})^{\rho\lambda}_{e}(\bar{Z}^{(1)e}_{\rho\lambda})_{B}+(\bar{Z}^{A}_{\mu\nu(V)})^{e}_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_{e})_{B}\\
={}&-\lambda\varepsilon_{\mu\nu\rho\lambda}\left(f^{A}_{aB}\frac{\delta S^{\mathrm{L}}}{\delta B_{a\rho\lambda}}+\frac{\partial f^{A}_{aB}}{\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c}_{\rho}}A^{a\lambda}\right)\\
&+\lambda\varepsilon_{\mu\nu\rho\lambda}\left(g^{AC}_{\ \ \ B}\frac{\delta S^{\mathrm{L}}}{\delta V^{C}_{\rho\lambda}}+\frac{\partial g^{AC}_{\ \ \ B}}{\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c}_{\rho}}V^{\lambda}_{C}\right),
\end{aligned}
\tag{C.15}
$$

$$
\begin{aligned}
&(\bar{Z}^{A}_{\mu\nu(V)})^{\sigma}_{B}(\bar{Z}^{(1)B}_{\sigma})^{\alpha\beta\gamma}_{a}+(\bar{Z}^{A}_{\mu\nu(V)})^{\rho\lambda}_{e}(\bar{Z}^{(1)e}_{\rho\lambda})^{\alpha\beta\gamma}_{a}+(\bar{Z}^{A}_{\mu\nu(V)})^{e}_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_{e})^{\alpha\beta\gamma}_{a}\\
={}&\tfrac{\lambda}{4!}\delta^{[\alpha}_{\mu}\delta^{\beta}_{\nu}\delta^{\gamma]}_{\rho}\frac{\partial f^{A}_{a}}{\partial\varphi_b}\frac{\delta S^{\mathrm{L}}}{\delta H^{b}_{\rho}},
\end{aligned}
\tag{C.16}
$$

$$
\begin{aligned}
&(\bar{Z}^{A}_{\mu\nu(V)})^{\sigma}_{B}(\bar{Z}^{(1)B}_{\sigma})^{a}_{\alpha\beta\gamma\delta}+(\bar{Z}^{A}_{\mu\nu(V)})^{\rho\lambda}_{e}(\bar{Z}^{(1)e}_{\rho\lambda})^{a}_{\alpha\beta\gamma\delta}\\
&+(\bar{Z}^{A}_{\mu\nu(V)})_{e}(\bar{Z}^{(1)e})^{a}_{\alpha\beta\gamma\delta}+(\bar{Z}^{A}_{\mu\nu(V)})^{e}_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_{e})^{a}_{\alpha\beta\gamma\delta}\\
={}&\tfrac{\lambda}{4!}\sigma_{\mu\mu'}\sigma_{\nu\nu'}\delta^{\mu'}_{[\alpha}\delta^{\nu'}_{\beta}\delta^{\rho}_{\gamma}\delta^{\lambda}_{\delta]}\left(f^{Aa}_{\ \ \ b}\frac{\delta S^{\mathrm{L}}}{\delta B^{\rho\lambda}_{b}}+\frac{\partial f^{Aa}_{\ \ \ b}}{\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c\rho}}A^{b}_{\lambda}\right)
\end{aligned}
$$

$$-\tfrac{\lambda}{4!}\sigma_{\mu\mu'}\sigma_{\nu\nu'}\delta^{\mu'}_{[\alpha}\delta^{\nu'}_{\beta}\delta^{\rho}_{\gamma}\delta^{\lambda}_{\delta]}\left(g^{aAB}\frac{\delta S^{\mathrm{L}}}{\delta V^{B\rho\lambda}}+\frac{\partial g^{aAB}}{\partial\varphi_b}\frac{\delta S^{\mathrm{L}}}{\delta H^{b\rho}}V_{B\lambda}\right), \quad \text{(C.17)}$$

$$\begin{aligned}
&(\bar{Z}^a_{\mu(H)})^e_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_e)^{\alpha\beta\gamma}_b+(\bar{Z}^a_{\mu(H)})^{\rho\lambda}_e(\bar{Z}^{(1)e}_{\rho\lambda})^{\alpha\beta\gamma}_b+(\bar{Z}^a_{\mu(H)})^{\sigma}_B(\bar{Z}^{(1)B}_{\sigma})^{\alpha\beta\gamma}_b\\
={}&\lambda\delta^{[\alpha}_{\mu}\delta^{\beta}_{\nu}\delta^{\gamma]}_{\rho}\left(\frac{\partial W_{bc}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta B_{c\nu\rho}}+\frac{\partial^2 W_{bc}}{\partial\varphi_a\partial\varphi_e}\frac{\delta S^{\mathrm{L}}}{\delta H^e_\nu}A^{c\rho}\right)\\
&-\tfrac{\lambda}{4!}\delta^{[\alpha}_{\mu}\delta^{\beta}_{\nu}\delta^{\gamma]}_{\rho}\left(\frac{\partial f^A_b}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta V^A_{\nu\rho}}+\frac{\partial^2 f^A_b}{\partial\varphi_a\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^c_\nu}V^\rho_A\right), \qquad \text{(C.18)}
\end{aligned}$$

$$\begin{aligned}
&(\bar{Z}^a_{\mu(H)})_e(\bar{Z}^{(1)e})^b_{\alpha\beta\gamma\delta}+(\bar{Z}^a_{\mu(H)})^e_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_e)^b_{\alpha\beta\gamma\delta}\\
&+(\bar{Z}^a_{\mu(H)})^{\rho\lambda}_e(\bar{Z}^{(1)e}_{\rho\lambda})^b_{\alpha\beta\gamma\delta}+(\bar{Z}^a_{\mu(H)})^{\sigma}_B(\bar{Z}^{(1)B}_{\sigma})^b_{\alpha\beta\gamma\delta}\\
={}&\tfrac{\lambda}{2}\sigma_{\mu[\alpha}\delta^{\nu}_{\beta}\delta^{\rho}_{\gamma}\delta^{\lambda}_{\delta]}\left(\frac{\partial M^b_{cd}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta B^{\nu\rho}_c}A^d_\lambda+\tfrac{1}{2}\frac{\partial M^b_{cd}}{\partial\varphi_a\partial\varphi_e}\frac{\delta S^{\mathrm{L}}}{\delta H^{e\nu}}A^c_\rho A^d_\lambda\right)\\
&+2\lambda\varepsilon_{\alpha\beta\gamma\delta}\left(\frac{\partial M^{bc}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta A^{c\mu}}+\frac{\partial^2 M^{bc}}{\partial\varphi_a\partial\varphi_d}\frac{\delta S^{\mathrm{L}}}{\delta H^d_\nu}B_{c\mu\nu}\right)\\
&-\tfrac{\lambda}{4!}\sigma_{\mu[\alpha}\delta^{\nu}_{\beta}\delta^{\rho}_{\gamma}\delta^{\lambda}_{\delta]}\left[\frac{\partial^2 f^{Ab}_{\ c}}{\partial\varphi_a\partial\varphi_d}\frac{\delta S^{\mathrm{L}}}{\delta H^{d\nu}}V_{A\rho}A^c_\lambda+\frac{\partial f^{Ab}_{\ c}}{\partial\varphi_a}\left(\frac{\delta S^{\mathrm{L}}}{\delta V^{A\nu\rho}}A^c_\lambda\right.\right.\\
&\left.\left.-\frac{\delta S^{\mathrm{L}}}{\delta B^{\nu\rho}_c}V_{A\lambda}\right)-\left(\frac{\partial^2 g^{bAB}}{\partial\varphi_a\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c\nu}}V_{A\rho}+\frac{\partial g^{bAB}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta V^{A\nu\rho}}\right)V_{B\lambda}\right], \qquad \text{(C.19)}
\end{aligned}$$

$$\begin{aligned}
&(\bar{Z}^a_{\mu(H)})^{\sigma}_C(\bar{Z}^{(1)C}_{\sigma})_A+(\bar{Z}^a_{\mu(H)})^{\rho\lambda}_e(\bar{Z}^{(1)e}_{\rho\lambda})_A+(\bar{Z}^a_{\mu(H)})^e_{\rho\lambda\sigma}(\bar{Z}^{(1)\rho\lambda\sigma}_e)_A\\
={}&\lambda\varepsilon_{\mu\nu\rho\lambda}\left[\frac{\delta S^{\mathrm{L}}}{\delta V^B_{\nu\rho}}\left(\frac{\partial f^B_{bA}}{\partial\varphi_a}A^{b\lambda}-\frac{\partial g^{BC}{}_A}{\partial\varphi_a}V^\lambda_C\right)-\frac{\partial f^B_{bA}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta B_{b\nu\rho}}V^\lambda_B\right.\\
&\left.+\frac{\delta S^{\mathrm{L}}}{\delta H^c_\nu}\left(\frac{\partial^2 f^B_{bA}}{\partial\varphi_a\partial\varphi_c}V^\rho_B A^{b\lambda}-\tfrac{1}{2}\frac{\partial^2 g^{BC}{}_A}{\partial\varphi_a\partial\varphi_c}V^\rho_B V^\lambda_C\right)\right]. \qquad \text{(C.20)}
\end{aligned}$$

The deformed gauge generators are given in (B.1)–(B.16) and S^{L} represents the deformed Lagrangian action (5.1).

The pieces of antighost number 3 from (4.111) that are simultaneously linear in the ghosts for ghosts for ghosts and in the antifields of the ghosts for ghosts offer us the second-order reducibility functions for the interacting model of the form

$$(\bar{Z}^{(2)A})^{\mu\nu\rho\lambda}_a=\tfrac{\lambda}{4!}f^A_a\varepsilon^{\mu\nu\rho\lambda}, \qquad \text{(C.21)}$$

$$(\bar{Z}^{(2)a}_{\alpha\beta\gamma})^{\mu\nu\rho\lambda}_{b} = -\tfrac{1}{6}\left(\delta^{a}_{b}\partial^{[\mu}\delta^{\nu}_{\alpha}\delta^{\rho}_{\beta}\delta^{\lambda]}_{\gamma} + \lambda\frac{\partial W_{cb}}{\partial\varphi_a}A^{c[\mu}\delta^{\nu}_{\alpha}\delta^{\rho}_{\beta}\delta^{\lambda]}_{\gamma}\right) + \tfrac{\lambda}{3!\cdot 4!}\delta^{[\mu}_{\alpha}\delta^{\nu}_{\beta}\delta^{\rho}_{\gamma}\delta^{\lambda]}_{\delta}\frac{\partial f^{A}_{b}}{\partial\varphi_a}V^{\delta}_{A}, \tag{C.22}$$

$$(\bar{Z}^{(2)\mu_1\mu_2\mu_3\mu_4}_{a})^{\mu\nu\rho\lambda}_{b} = \tfrac{\lambda}{12}W_{ab}\sum_{\pi\in S_4}(-)^{\pi}\,\sigma^{\mu_{\pi(1)}\mu}\sigma^{\mu_{\pi(2)}\nu}\sigma^{\mu_{\pi(3)}\rho}\sigma^{\mu_{\pi(4)}\lambda}, \tag{C.23}$$

where S_4 denotes the set of permutations of $\{1,2,3,4\}$ and $(-)^{\pi}$ is the signature of a given permutation π. By means of the terms with the antighost number equal to 3 present in (4.111) that are linear in the ghosts for ghosts for ghosts and also quadratic in antifields we infer the second-order reducibility relations for the interacting model in condensed De Witt form, which read as

$$\begin{aligned}&(\bar{Z}^{(1)A}_{\mu})_{B}(\bar{Z}^{(2)B})^{\alpha\beta\gamma\delta}_{a} + (\bar{Z}^{(1)A}_{\mu})^{\nu\rho\lambda}_{b}(\bar{Z}^{(2)b}_{\nu\rho\lambda})^{\alpha\beta\gamma\delta}_{a}\\ &+(\bar{Z}^{(1)A}_{\mu})^{b}_{\nu\rho\lambda\sigma}(\bar{Z}^{(2)\nu\rho\lambda\sigma}_{b})^{\alpha\beta\gamma\delta}_{a}\\ = {}& \tfrac{\lambda}{4!}\varepsilon^{\alpha\beta\gamma\delta}\frac{\partial f^{A}_{a}}{\partial\varphi_b}\frac{\delta S^{\mathrm{L}}}{\delta H^{b\mu}},\end{aligned} \tag{C.24}$$

$$\begin{aligned}&(\bar{Z}^{(1)\alpha\beta\gamma}_{a})_{A}(\bar{Z}^{(2)A})^{\mu\nu\rho\lambda}_{b} + (\bar{Z}^{(1)\alpha\beta\gamma}_{a})^{\delta\sigma\varepsilon}_{e}(\bar{Z}^{(2)e}_{\delta\sigma\varepsilon})^{\mu\nu\rho\lambda}_{b}\\ &+(\bar{Z}^{(1)\alpha\beta\gamma}_{a})^{e}_{\delta\sigma\varepsilon\eta}(\bar{Z}^{(2)\delta\sigma\varepsilon\eta}_{e})^{\mu\nu\rho\lambda}_{b}\\ = {}& \tfrac{\lambda}{3}\delta^{[\mu}_{\alpha'}\delta^{\nu}_{\beta'}\delta^{\rho}_{\gamma'}\delta^{\lambda]}_{\delta'}\sigma^{\alpha\alpha'}\sigma^{\beta\beta'}\sigma^{\gamma\gamma'}\frac{\partial W_{ab}}{\partial\varphi_c}\frac{\delta S^{\mathrm{L}}}{\delta H^{c}_{\delta'}},\end{aligned} \tag{C.25}$$

$$\begin{aligned}&(\bar{Z}^{(1)a}_{\mu\nu})_{A}(\bar{Z}^{(2)A})^{\alpha\beta\gamma\delta}_{b} + (\bar{Z}^{(1)a}_{\mu\nu})^{\delta\sigma\varepsilon}_{e}(\bar{Z}^{(2)e}_{\delta\sigma\varepsilon})^{\alpha\beta\gamma\delta}_{b}\\ &+(\bar{Z}^{(1)a}_{\mu\nu})^{e}_{\delta\sigma\varepsilon\eta}(\bar{Z}^{(2)\delta\sigma\varepsilon\eta}_{e})^{\alpha\beta\gamma\delta}_{b}\\ = {}& \tfrac{\lambda}{2}\delta^{[\alpha}_{\mu}\delta^{\beta}_{\nu}\delta^{\gamma}_{\rho}\delta^{\delta]}_{\lambda}\left[\frac{\delta S^{\mathrm{L}}}{\delta H^{d}_{\rho}}\left(\frac{\partial^2 W_{bc}}{\partial\varphi_a\partial\varphi_d}A^{c\lambda} - \tfrac{1}{4!}\frac{\partial^2 f^{A}_{b}}{\partial\varphi_a\partial\varphi_d}V^{\lambda}_{A}\right)\right.\\ &\left.+\frac{\partial W_{bc}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta B_{c\rho\lambda}} - \tfrac{1}{4!}\frac{\partial f^{A}_{b}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta V^{A}_{\rho\lambda}}\right].\end{aligned} \tag{C.26}$$

D Gauge algebra of the deformed model

The nonvanishing commutators among the deformed gauge transformations (5.7)–(5.12) result from the terms quadratic in the ghosts with pure ghost

number 1 present in (4.111). By analyzing these terms and taking into account the expressions (B.1)–(B.16), we deduce the following nonvanishing relations:

$$(\bar{Z}_{e(\varphi)})_b \frac{\delta(\bar{Z}_{a(\varphi)})_c}{\delta\varphi_e} - (\bar{Z}_{e(\varphi)})_c \frac{\delta(\bar{Z}_{a(\varphi)})_b}{\delta\varphi_e} = \lambda M^e_{bc} (\bar{Z}_{a(\varphi)})_e, \tag{D.1}$$

$$\begin{aligned} & (\bar{Z}_{e(\varphi)})_b \frac{\delta(\bar{Z}^a_{\mu(A)})_c}{\delta\varphi_e} + (\bar{Z}^m_{\sigma(A)})_b \frac{\delta(\bar{Z}^a_{\mu(A)})_c}{\delta A^m_\sigma} + (\bar{Z}^A_{\sigma(V)})_b \frac{\delta(\bar{Z}^a_{\mu(A)})_c}{\delta V^A_\sigma} \\ & -(\bar{Z}_{e(\varphi)})_c \frac{\delta(\bar{Z}^a_{\mu(A)})_b}{\delta\varphi_e} - (\bar{Z}^m_{\sigma(A)})_c \frac{\delta(\bar{Z}^a_{\mu(A)})_b}{\delta A^m_\sigma} - (\bar{Z}^A_{\sigma(V)})_c \frac{\delta(\bar{Z}^a_{\mu(A)})_b}{\delta V^A_\sigma} \\ = {} & \lambda \left[M^d_{bc} (\bar{Z}^a_{\mu(A)})_d + \tfrac{1}{12} M_{dbce} \varepsilon^{\alpha\beta\gamma\delta} A^e_\delta (\bar{Z}^a_{\mu(A)})^d_{\alpha\beta\gamma} \right. \\ & \left. -\tfrac{1}{3} f_{Abcd} \varepsilon^{\alpha\beta\gamma\delta} V^A_\delta (\bar{Z}^a_{\mu(A)})^d_{\alpha\beta\gamma} - \frac{\delta S^{\mathrm{L}}}{\delta H^{d\mu}} \frac{\partial M^a_{bc}}{\partial\varphi_d} \right], \end{aligned} \tag{D.2}$$

$$(\bar{Z}_{e(\varphi)})_b \frac{\delta(\bar{Z}^a_{\mu(A)})^c_{\alpha\beta\gamma}}{\delta\varphi_e} - (\bar{Z}^m_{\sigma(A)})^c_{\alpha\beta\gamma} \frac{\delta(\bar{Z}^a_{\mu(A)})_b}{\delta A^m_\sigma} = -\lambda M^c_{bd} (\bar{Z}^a_{\mu(A)})^d_{\alpha\beta\gamma}, \tag{D.3}$$

$$\begin{aligned} & (\bar{Z}_{e(\varphi)})_b \frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_c}{\delta\varphi_e} + (\bar{Z}^m_{\sigma(A)})_b \frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_c}{\delta A^m_\sigma} + (\bar{Z}^{\sigma\varepsilon}_{m(B)})_b \frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_c}{\delta B^{\sigma\varepsilon}_m} \\ & +(\bar{Z}^A_{\sigma(V)})_b \frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_c}{\delta V^A_\sigma} - (\bar{Z}_{e(\varphi)})_c \frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_b}{\delta\varphi_e} - (\bar{Z}^m_{\sigma(A)})_c \frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_b}{\delta A^m_\sigma} \\ & -(\bar{Z}^{\sigma\varepsilon}_{m(B)})_c \frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_b}{\delta B^{\sigma\varepsilon}_m} - (\bar{Z}^A_{\sigma(V)})_c \frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_b}{\delta V^A_\sigma} \\ = {} & \lambda \left\{ M^d_{bc} (\bar{Z}^{\mu\nu}_{a(B)})_d - \tfrac{1}{3} f_{Abcd} \varepsilon^{\alpha\beta\gamma\delta} V^A_\delta (\bar{Z}^{\mu\nu}_{a(B)})^d_{\alpha\beta\gamma} \right. \\ & +\tfrac{1}{12} M_{dbce} \varepsilon^{\alpha\beta\gamma\delta} A^e_\delta (\bar{Z}^{\mu\nu}_{a(B)})^d_{\alpha\beta\gamma} \\ & -\tfrac{1}{2} \left[\frac{\partial M^d_{bc}}{\partial\varphi_e} B_{d\alpha\beta} - \varepsilon_{\alpha\beta\gamma\delta} \left(\tfrac{1}{8} \frac{\partial M_{bcdf}}{\partial\varphi_e} A^{d\gamma} + \frac{\partial f^A_{bcf}}{\partial\varphi_e} V^\gamma_A \right) A^{f\delta} \right. \\ & \left. +\tfrac{1}{2} \varepsilon_{\alpha\beta\gamma\delta} \frac{\partial g^{AB}_{bc}}{\partial\varphi_e} V^\gamma_A V^\delta_B \right] (\bar{Z}^{\mu\nu}_{a(B)})^{\alpha\beta}_e + \left(g^{AB}_{bc} V_{B\lambda} - f^A_{bcd} A^d_\lambda \right) (\bar{Z}^{\mu\nu}_{a(B)})^\lambda_A \\ & -\lambda \varepsilon^{\mu\nu\rho\lambda} \left[\frac{\delta S^{\mathrm{L}}}{\delta H^{m\rho}} \left(\frac{\partial f^A_{abc}}{\partial\varphi_m} V_{A\lambda} - \tfrac{1}{4} \frac{\partial M_{abcd}}{\partial\varphi_m} A^d_\lambda \right) + f^A_{abc} \frac{\delta S^{\mathrm{L}}}{\delta V^{A\rho\lambda}} \right. \end{aligned}$$

$$
-\tfrac{1}{4}M_{abcd}\frac{\delta S^{\mathrm{L}}}{\delta B_d^{\rho\lambda}}\Bigg]\Bigg\}, \tag{D.4}
$$

$$
(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_c^{\alpha\beta}}{\delta\varphi_e}-(\bar{Z}^{\sigma\varepsilon}_{m(B)})_c^{\alpha\beta}\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_b}{\delta B_m^{\sigma\varepsilon}}=\lambda\frac{\partial W_{bc}}{\partial\varphi_d}(\bar{Z}^{\mu\nu}_{a(B)})_d^{\alpha\beta}, \tag{D.5}
$$

$$
\begin{aligned}
&(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})^c_{\alpha\beta\gamma}}{\delta\varphi_e}+(\bar{Z}^m_{\sigma(A)})_b\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})^c_{\alpha\beta\gamma}}{\delta A_\sigma^m}+(\bar{Z}^A_{\sigma(V)})_b\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})^c_{\alpha\beta\gamma}}{\delta V_\sigma^A}\\
&-(\bar{Z}^m_{\sigma(A)})^c_{\alpha\beta\gamma}\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_b}{\delta A_\sigma^m}-(\bar{Z}^{\sigma\varepsilon}_{m(B)})^c_{\alpha\beta\gamma}\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_b}{\delta B_m^{\sigma\varepsilon}}\\
=\;&\lambda\Bigg[-\tfrac{1}{4}\left(\frac{\partial M^c_{bd}}{\partial\varphi_e}A^d_{[\alpha}\delta^\rho_\beta\delta^\lambda_{\gamma]}+\tfrac{1}{12}\frac{\partial f^c_{Ab}}{\partial\varphi_e}V^A_{[\alpha}\delta^\rho_\beta\delta^\lambda_{\gamma]}\right)(\bar{Z}^{\mu\nu}_{a(B)})_{e\rho\lambda}\\
&+M^c_{eb}(\bar{Z}^{\mu\nu}_{a(B)})^e_{\alpha\beta\gamma}+\tfrac{1}{2}\frac{\partial M^c_{ab}}{\partial\varphi_m}\frac{\delta S^{\mathrm{L}}}{\delta H^{m\rho}}\delta^\mu_{[\alpha}\delta^\nu_\beta\delta^\rho_{\gamma]}\\
&+\tfrac{1}{4!}\varepsilon_{\lambda\alpha\beta\gamma}f_b^{\ Mc}(\bar{Z}^{\mu\nu}_{a(B)})^\lambda_M\Bigg],
\end{aligned} \tag{D.6}
$$

$$
\begin{aligned}
&(\bar{Z}^m_{\sigma(A)})^b_{\alpha\beta\gamma}\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})^c_{\alpha'\beta'\gamma'}}{\delta A_\sigma^m}-(\bar{Z}^m_{\sigma(A)})^c_{\alpha'\beta'\gamma'}\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})^b_{\alpha\beta\gamma}}{\delta A_\sigma^m}\\
=\;&-\tfrac{\lambda}{2}\frac{\partial M^{bc}}{\partial\varphi_e}\varepsilon^{\rho\lambda\delta\varepsilon}\varepsilon_{\delta\alpha\beta\gamma}\varepsilon_{\varepsilon\alpha'\beta'\gamma'}(\bar{Z}^{\mu\nu}_{a(B)})_{e\rho\lambda},
\end{aligned} \tag{D.7}
$$

$$
\begin{aligned}
&(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})^\lambda_A}{\delta\varphi_e}+(\bar{Z}^B_{\sigma(V)})_b\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})^\lambda_A}{\delta V_\sigma^B}-(\bar{Z}^{\sigma\varepsilon}_{m(B)})^\lambda_A\frac{\delta(\bar{Z}^{\mu\nu}_{a(B)})_b}{\delta B_m^{\sigma\varepsilon}}\\
=\;&-\lambda f^B_{bA}(\bar{Z}^{\mu\nu}_{a(B)})^\lambda_B+\tfrac{\lambda}{2}\varepsilon^{\alpha\beta\rho\lambda}\frac{\partial f_{bMA}}{\partial\varphi_e}V^M_\rho(\bar{Z}^{\mu\nu}_{a(B)})_{e\alpha\beta},
\end{aligned} \tag{D.8}
$$

$$
\begin{aligned}
&(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^A_{\mu(V)})_c}{\delta\varphi_e}+(\bar{Z}^B_{\sigma(V)})_b\frac{\delta(\bar{Z}^A_{\mu(V)})_c}{\delta V_\sigma^B}-(\bar{Z}_{e(\varphi)})_c\frac{\delta(\bar{Z}^A_{\mu(V)})_b}{\delta\varphi_e}\\
&-(\bar{Z}^B_{\sigma(V)})_c\frac{\delta(\bar{Z}^A_{\mu(V)})_b}{\delta V_\sigma^B}=\lambda M^d_{bc}(\bar{Z}^A_{\mu(V)})_d,
\end{aligned} \tag{D.9}
$$

$$
(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_c}{\delta\varphi_e}+(\bar{Z}^m_{\sigma(A)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_c}{\delta A_\sigma^m}+(\bar{Z}^{\sigma\varepsilon}_{m(B)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_c}{\delta B_m^{\sigma\varepsilon}}
$$

$$
\begin{aligned}
&+(\bar{Z}^B_{\sigma(V)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_c}{\delta V^B_\sigma}+(\bar{Z}^B_{\sigma\varepsilon(V)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_c}{\delta V^B_{\sigma\varepsilon}}-(\bar{Z}_{e(\varphi)})_c\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta\varphi_e}\\
&-(\bar{Z}^m_{\sigma(A)})_c\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta A^m_\sigma}-(\bar{Z}^{\sigma\varepsilon}_{m(B)})_c\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta B^{\sigma\varepsilon}_m}-(\bar{Z}^B_{\sigma(V)})_c\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta V^B_\sigma}\\
&-(\bar{Z}^B_{\sigma\varepsilon(V)})_c\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta V^B_{\sigma\varepsilon}}\\
=\;&\lambda\left\{M^d_{bc}(\bar{Z}^A_{\mu\nu(V)})_d-\tfrac{1}{3}\varepsilon^{\alpha\beta\gamma\delta}\left[f_{Mbcd}V^M_\delta-\tfrac{1}{4}M_{dbce}A^e_\delta\right](\bar{Z}^A_{\mu\nu(V)})^d_{\alpha\beta\gamma}\right.\\
&-\tfrac{1}{2}\left[\frac{\partial M^d_{bc}}{\partial\varphi_e}B_{d\alpha\beta}-\varepsilon_{\alpha\beta\gamma\delta}\left(\tfrac{1}{8}\frac{\partial M_{bcdf}}{\partial\varphi_e}A^{d\gamma}+\frac{\partial f^M_{bcf}}{\partial\varphi_e}V^\gamma_M\right)A^{f\delta}\right.\\
&\left.+\tfrac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\frac{\partial g^{BC}_{bc}}{\partial\varphi_e}V^\gamma_BV^\delta_C\right](\bar{Z}^A_{\mu\nu(V)})^{\alpha\beta}_e+\left(g^{MB}_{bc}V_{B\lambda}-f^M_{bcd}A^d_\lambda\right)(\bar{Z}^A_{\mu\nu(V)})^\lambda_M\\
&+\varepsilon_{\mu\nu\rho\lambda}\left[\frac{\delta S^{\mathrm{L}}}{\delta H^m_\rho}\left(\frac{\partial f^A_{bcd}}{\partial\varphi_m}A^{d\lambda}-\frac{\partial g^{AB}_{bc}}{\partial\varphi_m}V^\lambda_B\right)+f^A_{bcd}\frac{\delta S^{\mathrm{L}}}{\delta B_{d\rho\lambda}}\right.\\
&\left.\left.-g^{AB}_{bc}\frac{\delta S^{\mathrm{L}}}{\delta V^B_{\rho\lambda}}\right]\right\},
\end{aligned}
\tag{D.10}
$$

$$
\begin{aligned}
&(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^c_{\alpha\beta\gamma}}{\delta\varphi_e}+(\bar{Z}^m_{\sigma(A)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^c_{\alpha\beta\gamma}}{\delta A^m_\sigma}+(\bar{Z}^B_{\sigma(V)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^c_{\alpha\beta\gamma}}{\delta V^B_\sigma}\\
&-(\bar{Z}^m_{\sigma(A)})^c_{\alpha\beta\gamma}\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta A^m_\sigma}-(\bar{Z}^{\sigma\varepsilon}_{m(B)})^c_{\alpha\beta\gamma}\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta B^{\sigma\varepsilon}_m}-(\bar{Z}^B_{\sigma\varepsilon(V)})^c_{\alpha\beta\gamma}\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta V^B_{\sigma\varepsilon}}\\
=\;&\lambda\left[-\tfrac{1}{4}\left(\frac{\partial M^c_{bd}}{\partial\varphi_e}A^d_{[\alpha}\delta^\rho_\beta\delta^\lambda_{\gamma]}+\tfrac{1}{12}\frac{\partial f^c_{Mb}}{\partial\varphi_e}V^M_{[\alpha}\delta^\rho_\beta\delta^\lambda_{\gamma]}\right)(\bar{Z}^A_{\mu\nu(V)})_{e\rho\lambda}\right.\\
&+M^c_{eb}(\bar{Z}^A_{\mu\nu(V)})^e_{\alpha\beta\gamma}+\tfrac{1}{4!}\varepsilon_{\lambda\alpha\beta\gamma}f^{Mc}{}_b(\bar{Z}^A_{\mu\nu(V)})^\lambda_M\\
&\left.-\tfrac{1}{4!}\varepsilon_{\mu\nu\rho\lambda}\varepsilon_{\sigma\alpha\beta\gamma}\sigma^{\lambda\sigma}\frac{\partial f^{Ac}{}_b}{\partial\varphi_m}\frac{\delta S^{\mathrm{L}}}{\delta H^m_\rho}\right],
\end{aligned}
\tag{D.11}
$$

$$
\begin{aligned}
&(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^{\alpha\beta}_c}{\delta\varphi_e}-(\bar{Z}^B_{\sigma\varepsilon(V)})^{\alpha\beta}_c\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta V^B_{\sigma\varepsilon}}-(\bar{Z}^{\sigma\varepsilon}_{m(B)})^{\alpha\beta}_c\frac{\delta(\bar{Z}_{\mu\nu(V)})_b}{\delta B^{\sigma\varepsilon}_m}\\
=\;&\lambda\frac{\partial W_{bc}}{\partial\varphi_d}(\bar{Z}^A_{\mu\nu(V)})^{\alpha\beta}_d,
\end{aligned}
\tag{D.12}
$$

$$
(\bar{Z}^m_{\sigma(A)})^b_{\alpha\beta\gamma}\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^c_{\alpha'\beta'\gamma'}}{\delta A^m_\sigma}-(\bar{Z}^m_{\sigma(A)})^c_{\alpha'\beta'\gamma'}\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^b_{\alpha\beta\gamma}}{\delta A^m_\sigma}
$$

$$= \; -\tfrac{\lambda}{2}\frac{\partial M^{bc}}{\partial\varphi_e}\varepsilon^{\rho\lambda\delta\varepsilon}\varepsilon_{\delta\alpha\beta\gamma}\varepsilon_{\varepsilon\alpha'\beta'\gamma'}(\bar{Z}^A_{\mu\nu(V)})_{e\rho\lambda}, \tag{D.13}$$

$$\begin{aligned}
&(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^\lambda_B}{\delta\varphi_e}+(\bar{Z}^m_{\sigma(A)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^\lambda_B}{\delta A^m_\sigma}+(\bar{Z}^C_{\sigma(V)})_b\frac{\delta(\bar{Z}^A_{\mu\nu(V)})^\lambda_B}{\delta V^C_\sigma}\\
&-(\bar{Z}^{\sigma\varepsilon}_{m(B)})^\lambda_B\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta B^{\sigma\varepsilon}_m}-(\bar{Z}^C_{\sigma\varepsilon(V)})^\lambda_B\frac{\delta(\bar{Z}^A_{\mu\nu(V)})_b}{\delta V^C_{\sigma\varepsilon}}\\
= \;&-\lambda f^M_{bB}(\bar{Z}^A_{\mu\nu(V)})^\lambda_M+\tfrac{\lambda}{2}\varepsilon^{\alpha\beta\rho\lambda}\frac{\partial f_{bMB}}{\partial\varphi_e}V^M_\rho(\bar{Z}^A_{\mu\nu(V)})_{e\alpha\beta}\\
&+\lambda\sigma^{\lambda\sigma}\varepsilon_{\mu\nu\rho\sigma}\frac{\partial f^A_{bB}}{\partial\varphi_m}\frac{\delta S^{\mathrm{L}}}{\delta H^m_\rho},
\end{aligned} \tag{D.14}$$

$$\begin{aligned}
&(\bar{Z}_{e(\varphi)})_b\frac{\delta(\bar{Z}^a_{\mu(H)})_c}{\delta\varphi_e}+(\bar{Z}^m_{\sigma(A)})_b\frac{\delta(\bar{Z}^a_{\mu(H)})_c}{\delta A^m_\sigma}+(\bar{Z}^m_{\sigma(H)})_b\frac{\delta(\bar{Z}^a_{\mu(H)})_c}{\delta H^m_\sigma}\\
&+(\bar{Z}^{\sigma\varepsilon}_{m(B)})_b\frac{\delta(\bar{Z}^a_{\mu(H)})_c}{\delta B^{\sigma\varepsilon}_m}+(\bar{Z}^A_{\sigma(V)})_b\frac{\delta(\bar{Z}^a_{\mu(H)})_c}{\delta V^A_\sigma}+(\bar{Z}^A_{\sigma\varepsilon(V)})_b\frac{\delta(\bar{Z}^a_{\mu(H)})_c}{\delta V^A_{\sigma\varepsilon}}\\
&-(\bar{Z}_{e(\varphi)})_c\frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta\varphi_e}-(\bar{Z}^m_{\sigma(A)})_c\frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta A^m_\sigma}-(\bar{Z}^m_{\sigma(H)})_c\frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta H^m_\sigma}\\
&-(\bar{Z}^{\sigma\varepsilon}_{m(B)})_c\frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta B^{\sigma\varepsilon}_m}-(\bar{Z}^A_{\sigma(V)})_c\frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta V^A_\sigma}-(\bar{Z}^A_{\sigma\varepsilon(V)})_c\frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta V^A_{\sigma\varepsilon}}\\
= \;&\lambda\Big\{M^d_{bc}(\bar{Z}^a_{\mu(H)})_d-\tfrac{1}{3}\varepsilon^{\alpha\beta\gamma\delta}\left[f_{Mbcd}V^M_\delta-\tfrac{1}{4}M_{dbce}A^e_\delta\right](\bar{Z}^a_{\mu(H)})^d_{\alpha\beta\gamma}\\
&-\tfrac{1}{2}\left[\frac{\partial M^d_{bc}}{\partial\varphi_e}B_{d\alpha\beta}-\varepsilon_{\alpha\beta\gamma\delta}\left(\tfrac{1}{8}\frac{\partial M_{bcdf}}{\partial\varphi_e}A^{d\gamma}+\frac{\partial f^M_{bcf}}{\partial\varphi_e}V^\gamma_M\right)A^{f\delta}\right.\\
&\left.+\tfrac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\frac{\partial g^{BC}_{bc}}{\partial\varphi_e}V^\gamma_BV^\delta_C\right](\bar{Z}^a_{\mu(H)})^{\alpha\beta}_e+\left(g^{MB}_{bc}V_{B\lambda}-f^M_{bcd}A^d_\lambda\right)(\bar{Z}^a_{\mu(H)})^\lambda_M\\
&+\frac{\delta S^{\mathrm{L}}}{\delta H^m_\nu}\left[\frac{\partial^2M^d_{bc}}{\partial\varphi_m\partial\varphi_a}B_{d\mu\nu}+\tfrac{1}{2}\varepsilon_{\mu\nu\rho\lambda}\left(\frac{\partial^2g^{AB}_{bc}}{\partial\varphi_m\partial\varphi_a}V^\rho_AV^\lambda_B\right.\right.\\
&\left.\left.-2\frac{\partial^2f^A_{bcd}}{\partial\varphi_m\partial\varphi_a}V^\rho_AA^{d\lambda}-\tfrac{1}{4}\frac{\partial^2M_{bcde}}{\partial\varphi_m\partial\varphi_a}A^{d\rho}A^{e\lambda}\right)\right]\\
&+\varepsilon_{\mu\nu\rho\lambda}\frac{\delta S^{\mathrm{L}}}{\delta B_{d\rho\lambda}}\left(\frac{\partial f^A_{bcd}}{\partial\varphi_a}V^\nu_A-\tfrac{1}{8}\frac{\partial M_{bcde}}{\partial\varphi_a}A^{e\nu}\right)\\
&+\varepsilon_{\mu\nu\rho\lambda}\frac{\delta S^{\mathrm{L}}}{\delta V^A_{\rho\lambda}}\left(\frac{\partial g^{AB}_{bc}}{\partial\varphi_a}V^\nu_B-\frac{\partial f^A_{bcd}}{\partial\varphi_a}A^{d\nu}\right)+\frac{\partial M^d_{bc}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta A^{d\mu}}\Big\},
\end{aligned} \tag{D.15}$$

$$
\begin{aligned}
& (\bar Z_{e(\varphi)})_b \frac{\delta(\bar Z^a_{\mu(H)})^c_{\alpha\beta\gamma}}{\delta\varphi_e} + (\bar Z^m_{\sigma(A)})_b \frac{\delta(\bar Z^a_{\mu(H)})^c_{\alpha\beta\gamma}}{\delta A^m_\sigma} + (\bar Z^{\sigma\varepsilon}_{m(B)})_b \frac{\delta(\bar Z^a_{\mu(H)})^c_{\alpha\beta\gamma}}{\delta B^{\sigma\varepsilon}_m} \\
& + (\bar Z^A_{\varepsilon(V)})_b \frac{\delta(\bar Z^a_{\mu(H)})^c_{\alpha\beta\gamma}}{\delta V^A_\varepsilon} - (\bar Z^m_{\sigma(A)})^c_{\alpha\beta\gamma} \frac{\delta(\bar Z^a_{\mu(H)})_b}{\delta A^m_\sigma} \\
& - (\bar Z^m_{\sigma(H)})^c_{\alpha\beta\gamma} \frac{\delta(\bar Z^a_{\mu(H)})_b}{\delta H^m_\sigma} - (\bar Z^{\sigma\varepsilon}_{m(B)})^c_{\alpha\beta\gamma} \frac{\delta(\bar Z^a_{\mu(H)})_b}{\delta B^{\sigma\varepsilon}_m} \\
= \; & \lambda \Bigg\{ -\tfrac{1}{4} \left(\frac{\partial M^c_{bd}}{\partial\varphi_e} A^d_{[\alpha}\delta^\rho_\beta\delta^\lambda_{\gamma]} + \tfrac{1}{12} \frac{\partial f^c_{Ab}}{\partial\varphi_e} V^A_{[\alpha}\delta^\rho_\beta\delta^\lambda_{\gamma]} \right) (\bar Z^a_{\mu(H)})_{e\rho\lambda} \\
& + M^c_{eb} (\bar Z^a_{\mu(H)})^e_{\alpha\beta\gamma} + \tfrac{1}{4!} \varepsilon_{\lambda\alpha\beta\gamma} f^{Mc}_{\ \ b} (\bar Z^a_{\mu(H)})^\lambda_M \\
& + \tfrac{1}{2} \sigma_{\mu[\alpha} \delta^\nu_\beta \delta^\rho_{\gamma]} \Bigg[\frac{\delta S^{\mathrm{L}}}{\delta H^{m\nu}} \left(\frac{\partial^2 M^c_{bd}}{\partial\varphi_a\partial\varphi_m} A^d_\rho + \tfrac{1}{12} \frac{\partial^2 f^c_{Ab}}{\partial\varphi_a\partial\varphi_m} V^A_\rho \right) \\
& + \frac{\partial M^c_{bd}}{\partial\varphi_a} \frac{\delta S^{\mathrm{L}}}{\delta B^{\nu\rho}_d} + \tfrac{1}{12} \frac{\partial f^c_{Ab}}{\partial\varphi_a} \frac{\delta S^{\mathrm{L}}}{\delta V^{\nu\rho}_A} \Bigg] \Bigg\},
\end{aligned}
\tag{D.16}
$$

$$
\begin{aligned}
& (\bar Z_{e(\varphi)})_b \frac{\delta(\bar Z^a_{\mu(H)})^{\alpha\beta}_c}{\delta\varphi_e} + (\bar Z^m_{\sigma(A)})_b \frac{\delta(\bar Z^a_{\mu(H)})^{\alpha\beta}_c}{\delta A^m_\sigma} + (\bar Z^A_{\sigma(V)})_b \frac{\delta(\bar Z^a_{\mu(H)})^{\alpha\beta}_c}{\delta V^A_\sigma} \\
& - (\bar Z^m_{\sigma(H)})^{\alpha\beta}_c \frac{\delta(\bar Z^a_{\mu(H)})_b}{\delta H^m_\sigma} - (\bar Z^{\sigma\varepsilon}_{m(B)})^{\alpha\beta}_c \frac{\delta(\bar Z^a_{\mu(H)})_b}{\delta B^{\sigma\varepsilon}_m} - (\bar Z^A_{\sigma\varepsilon(V)})^{\alpha\beta}_c \frac{\delta(\bar Z^a_{\mu(H)})_b}{\delta V^A_{\sigma\varepsilon}} \\
= \; & \lambda \left[\frac{\partial W_{bc}}{\partial\varphi_d} (\bar Z^a_{\mu(H)})^{\alpha\beta}_d - \frac{\partial^2 W_{bc}}{\partial\varphi_a\partial\varphi_d} \frac{\delta S^{\mathrm{L}}}{\delta H^d_\nu} \delta^{[\alpha}_\mu \delta^{\beta]}_\nu \right],
\end{aligned}
\tag{D.17}
$$

$$
\begin{aligned}
& (\bar Z^m_{\sigma(A)})^b_{\alpha\beta\gamma} \frac{\delta(\bar Z^a_{\mu(H)})^c_{\alpha'\beta'\gamma'}}{\delta A^m_\sigma} + (\bar Z^{\sigma\varepsilon}_{m(B)})^b_{\alpha\beta\gamma} \frac{\delta(\bar Z^a_{\mu(H)})^c_{\alpha'\beta'\gamma'}}{\delta B^{\sigma\varepsilon}_m} \\
& - (\bar Z^m_{\sigma(A)})^c_{\alpha'\beta'\gamma'} \frac{\delta(\bar Z^a_{\mu(H)})^b_{\alpha\beta\gamma}}{\delta A^m_\sigma} - (\bar Z^{\sigma\varepsilon}_{m(B)})^c_{\alpha'\beta'\gamma'} \frac{\delta(\bar Z^a_{\mu(H)})^b_{\alpha\beta\gamma}}{\delta B^{\sigma\varepsilon}_m} \\
= \; & -\tfrac{\lambda}{2} \frac{\partial M^{bc}}{\partial\varphi_e} \varepsilon^{\rho\lambda\delta\varepsilon} \varepsilon_{\delta\alpha\beta\gamma} \varepsilon_{\varepsilon\alpha'\beta'\gamma'} (\bar Z^a_{\mu(H)})_{e\rho\lambda} \\
& + \lambda \varepsilon_{\mu\nu\rho\lambda} \varepsilon_{\delta\alpha\beta\gamma} \varepsilon_{\varepsilon\alpha'\beta'\gamma'} \sigma^{\rho\delta} \sigma^{\varepsilon\lambda} \frac{\partial^2 M^{bc}}{\partial\varphi_a\partial\varphi_d} \frac{\delta S^{\mathrm{L}}}{\delta H^d_\nu},
\end{aligned}
\tag{D.18}
$$

$$
(\bar Z_{e(\varphi)})_b \frac{\delta(\bar Z^a_{\mu(H)})^\lambda_B}{\delta\varphi_e} + (\bar Z^m_{\sigma(A)})_b \frac{\delta(\bar Z^a_{\mu(H)})^\lambda_B}{\delta A^m_\sigma} + (\bar Z^C_{\sigma(V)})_b \frac{\delta(\bar Z^a_{\mu(H)})^\lambda_B}{\delta V^C_\sigma}
$$

$$
\begin{aligned}
& -(\bar{Z}^C_{\sigma\varepsilon(V)})^\lambda_B \frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta V^C_{\sigma\varepsilon}} - (\bar{Z}^m_{\sigma(H)})^\lambda_B \frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta H^m_\sigma} - (\bar{Z}^{\sigma\varepsilon}_{m(B)})^\lambda_B \frac{\delta(\bar{Z}^a_{\mu(H)})_b}{\delta B^{\sigma\varepsilon}_m} \\
= \; & -\lambda f^M_{bB}(\bar{Z}^a_{\mu(H)})^\lambda_M + \tfrac{\lambda}{2}\varepsilon^{\alpha\beta\rho\lambda}\frac{\partial f_{bMB}}{\partial\varphi_e}V^M_\rho(\bar{Z}^a_{\mu(H)})_{e\alpha\beta} \\
& -\lambda\varepsilon_{\mu\nu\rho\sigma}\sigma^{\sigma\lambda}\left(\frac{\partial^2 f_{bMB}}{\partial\varphi_a\partial\varphi_e}V^{M\rho}\frac{\delta S^{\mathrm{L}}}{\delta H^e_\nu} + \frac{\partial f_{bMB}}{\partial\varphi_a}\frac{\delta S^{\mathrm{L}}}{\delta V_{M\nu\rho}}\right). \qquad \text{(D.19)}
\end{aligned}
$$

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